In preparing for the Naval Reactors (NR) interview, each midshipman will be his own best judge as to what areas he needs to review. In every case, it is important to prepare a systematic plan of attack in order to be fully prepared for the interview. Formulating, adhering to, and following through on this regimen will give him the confidence he needs to do well at the interviews. Here are a few points that should be helpful in developing a study regimen:

1) SET ASIDE A CONSISTENT TIME TO STUDY.

Essential to each midshipman’s success is the investment of time in studying. The best way to maximize time and effort is to set aside a given quantity of time on a daily basis to study. This will help in three ways. First, this will minimize the chances of forgetting subject matter previously covered. Secondly, this sets up a daily format which allows for an orderly progression in one’s studies. Finally, this will combat procrastination in the sense that the student will better be able to plan time for other school work since he will have a specific time set aside for review.

In determining the amount of time, it is important to keep in mind that marathon sessions will tend to become discouraging after a time. A regular blocked time of one to two hours on set days of the week should prove sufficient, as well as not become overly discouraging. The study regimen should not be such that it detracts from a midshipman’s regular studies. As mentioned above, the midshipman can best judge how much time he will actually need. The key is to make his preparations a part of his "routine".

2) WORK WITH A PARTNER

Working with someone else in preparing for the interview will prove to be extremely valuable for several reasons. First, each individual will become accountable to someone else. It is not as easy to ignore one’s review when someone else is depending on him. Also, each will have an additional source of knowledge with which to share and receive knowledge concerning formulae, ways to attack problems, and alternative answers to questions. Finally, it gives the individual someone to verbally work out problems with as he will have to do at his NR interview. Thinking that he knows how a problem is worked and actually discussing why a problem is worked the way it is will acutely demonstrate what areas need additional attention.

3) PAY PARTICULAR ATTENTION TO PHYSICS AND CALCULUS.

In a review of previous questions asked at NR interviews, there appeared to be a significant consistency with which Physics and Calculus questions were asked of all majors. It is almost a certainty that a midshipman will be asked some form of Physics problem (i.e., Statics, Dynamics, etc.) as well as a few Calculus problems (i.e., Derivatives, Integrals, etc.). Therefore, it would behoove each midshipman to bring himself up to a high level of proficiency in
these areas. This is not to say that one should totally ignore his chosen major in lieu of Physics and Calculus; rather, he should be prepared to fully answer questions in the areas of Physics and Calculus as well as in his major area of study.

The following is a general guideline* to aid midshipmen in determining which areas they might need review in addition to their major, if time permits:

**NON-TECHNICAL MAJORS** - Calculus, Differential Equations (if taken), Graphing, Physics (all areas covered in the first two semesters of calculus-based physics).

**NON-ENGINEERING TECHNICAL MAJORS** - (PHYSICS, BIOLOGY, CHEMISTRY, MATH) -

**Math** - General, Graphing, Calculus (differential and integral), Differential Equations, Probability and Statistics.


**Chemistry** - primarily pH and Bohr model (for non-Chem majors).

**ENGINEERING MAJORS (ALL TYPES)** - all Math, Basic Physics, Advanced Physics if applicable, respective major, and other technical courses taken as applicable.

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4) **WORK PROBLEMS.**

The major pitfall for most interviewees is inadequate preparation. Too often people will glance over the review guide, thinking they know how to work the problem as well as what the answer should be. At the interviews, though, they discover that what they thought they knew was wrong or not fully developed. This was a direct result of not putting pencil to paper and working out the problems.

Working out the problems will allow one to achieve an understanding of the problems. In doing so, it is important to realize why he is working a problem in a given way as well as how. Case in point: given a projectile motion problem, one might immediately employ the equation \( x = x_0 + v_x t + (a_y t^2)/2 \) in order to show how to work the problem. What would one do if asked why the equation is appropriate? Would he be able to show how this equation was derived? Interviewees often stumble at interviews if they cannot explain from where an equation came.

A second procedure that will benefit a midshipman is to practice working out the problems as if he were explaining them to someone else (this is easily done if he has a study partner). Explaining what he is doing as he works out the problem is part of the format at interviews. Practicing this beforehand will give him an extra measure of confidence, enhancing his chances of selection.
5) GO OVER PERSONAL QUESTIONS.

An important aspect of the interview is the way one handles personal questions. Again, it is almost certain that a midshipman will be asked one or two such questions, usually at the beginning of the interview. A well thought-out answer to a given question will not, at the very least, reflect negatively on him. Conversely, a good answer may make a good first impression with his interviewer; and starting off on the right foot will improve his chances of a favorable interview. Therefore, it is important not to neglect this aspect of his preparation. He should review the personal questions in this guide and think about them as they apply to him. He should rehearse how he would answer them and try to foresee what questions might follow from the answer to the original question. He should review his transcript and anticipate what questions might be asked about a particular course, a semester GPA, etc. Well thought-out answers to these possible questions will stand him in good stead with the interviewer, and especially with Admiral McKee. Possible answers to some questions might be in the CNET question and answer booklet concerning nuclear power.

6) BE POSITIVE

A midshipman's ability to project his character and personality will be dependent on his level of confidence. If he prepares as advised in the above paragraphs, he will feel confident prior to the interviews. One final way in which he can raise his confidence level is to mentally prepare himself. He can do this in two ways. First, he can take advantage of the interview discussion in this review guide. He will be better prepared to face whatever circumstances arise if he knows what lies ahead and can visualize how he will handle different situations. Also, he can learn to operate under pressure during the practice interviews. Adjusting to the pressure of such an interview and correcting his mistakes during it will enable him to perform efficiently and confidently at interviews.

7) PRACTICE INTERVIEWS.

Next to actually working out the problems, this will be a midshipman's most valuable preparation step. Actually being asked to answer questions and explain problems allows him to get a realistic feeling of what to expect. These practice interviews will often expose a weak area of study prior to the NR interview, giving him the time to shore up these areas. Faltering and becoming flustered during the practice interviews are common experiences among interviewees. He should not get discouraged if this happens; instead, he should learn from it. Why did he become flustered? What area was it in? Was it because he was not prepared or was it that he needed more exposure to the interviewing environment? Determine why, and he will be less likely to become flustered again. The practice interview also leads to the realization that working a problem on paper and explaining it to an NR staff member, under a time constraint, can be two very different situations. The pressure one feels during an interview may radically alter his thought processes and his ability to verbalize his ideas. A proven way to determine how one performs under pressure is through a practice interview. Finally, it is recommended that each midshipman be interviewed by at
least two different individuals, such as the unit's NPO and CO. This will allow a slightly different emphasis to be placed upon the questions asked, thus creating an environment closer to that existing at NR.

8) A FINAL WORD.

A study regimen encompassing all of the above will more than adequately prepare one for the interview. As was said above, the individual midshipman will be the best judge as to what he needs to study. Keep in mind, however, that he will only have thirty minutes with each interviewer. That means he will have thirty minutes to show his intelligence, personality, and character makeup. A study regimen will allow him to do this.

* - This guide is based on the debrief sheets obtained from interviewees for FY87. It is not a guarantee that the questions asked in FY88 and subsequent years will follow the same trend. It is only meant to give one a starting point in ascertaining what we should study.
EXAMPLES OF INTERVIEW QUESTIONS OF A PERSONAL NATURE

1. What do you like to do in your spare time?
2. Do you have any marriage plans?
3. Do you think your grades accurately reflect your knowledge in the subjects taken?
4. What is your biggest problem?
5. What do you see yourself doing in 20 years?
6. What accomplishments have made you proud?
7. If you could change your major, what would you have taken?
8. What books have you read in the past year?
9. What is the most outstanding thing you've ever done?
10. What is the toughest/easiest class you've taken?
11. What has been your biggest challenge to date?
12. How did your earlier interviews go?
13. Why do you want to be a part of the nuclear propulsion program?
14. Why should you be selected for the program?
15. Why did you choose the school that you did?
16. Why did you choose the major that you did?
17. How much time per week did you study?
18. What were some of your high school activities?
19. What were some of your college activities?
20. Why did you receive such a low grade in this class?
21. Why did you receive such a low grade during this semester?
22. Tell me about your prior enlisted service.
23. What has been some of your previous work experiences?
24. Do you want surface or subs? Why?
25. What are you doing in your current courses?
26. What assurance can you give that you will successfully complete nuclear power school (NPS)?

27. What are the hours of study required in your major as compared to the number required of a engineering (non-tech) major at your school?

28. Why were you so nervous in interviews?

29. How did you pay for college?

30. What did you do during the unaccounted-for time on your transcript?

31. To what other schools did you apply?

32. In what extracurricular activities did you participate?

33. Discuss any summer jobs and school projects?

34. Is your school accredited?

35. How did your school/dept. compare with others?

36. Why are you a technical major when you do so much better in non-technical subjects?

37. Are you fully aware of what you will undergo at NPS?

38. How were you informed about NPS?

39. Do you feel that your preparation was adequate enough to get you selected?
1. **NEWTON'S FIRST LAW:** A body will continue in its state of rest, or in uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it.

\[ \Sigma F_x = 0, \Sigma F_y = 0 \] (first condition of equilibrium)

2. **NEWTON'S SECOND LAW:** The change of motion is proportional to the motive force impressed, and is made in the direction of the straight line in which that force is impressed.

\[ \Sigma F_x = ma_x, \Sigma F_y = ma_y \]

3. **NEWTON'S THIRD LAW:** To every action there is always opposed an equal reaction, or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.

4. **FRICTION:** Static Force \( F_S = \mu_s N \), Kinetic force \( F_K = \mu_k N \)

5. **MOMENT:** The product of the magnitude of a force and its force arm.

\[ \Sigma M = 0 \] (about any arbitrary axis) (second condition of equilibrium)

6. **VELOCITY:** \( v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \)

The instantaneous velocity at any point of a coordinate-time graph equals the slope of the tangent to the graph at that point.

7. **ACCELERATION:** \( a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{v}{dx} \)

The instantaneous acceleration at any point of a velocity-time graph equals the slope of the tangent to the graph at that point.
8. **RECTILINEAR MOTION** with constant acceleration

\[ v = v_0 + at \quad \Rightarrow \quad v^2 = v_0^2 + 2ax \]

\[ x = (v_0 + v)t/2 \quad \Rightarrow \quad x = v_0 t + (1/2)at^2 \]

9. **VELOCITY AND COORDINATE BY INTEGRATION**

\[ v = \int a(t) \, dt + c_1 \quad \text{where} \quad c_1 \text{ expressed in terms of } v_0 \text{ when } t=0 \]

\[ x = \int v(t) \, dt + c_2 \quad \text{where} \quad c_2 \text{ expressed in terms of } x_0 \text{ when } t=0 \]

\[ v^2/2 = \int a(x) \, dx + c_3 \quad \text{where} \quad c_3 \text{ expressed in terms of } v_0 \text{ when } x=0 \]

10. **MOTION OF A PROJECTILE**

\[ v = \sqrt{v_x^2 + v_y^2} \quad \text{where} \quad v_x = v_0 \cos \theta_0 \]

\[ v_y = v_0 \sin \theta_0 - gt \]

\[ x = (v_0 \cos \theta_0)t \quad y = (v_0 \sin \theta_0)t - gt^2/2 \]

\[ \theta_0 = 45 \text{ degrees elevation gives maximum range.} \]

11. **CIRCULAR MOTION**

\[ v = 2\pi R/T \quad \text{where} \quad T = \text{time to complete one revolution} \]

\[ a_R = v^2/R \quad \text{and} \quad F = mv^2/R \]

**Centripetal Force:** Direction of resultant force is toward center.

Forces on a body swinging in a vertical circle:
Resultant Acceleration:

\[ a = \sqrt{a_T^2 + a_R^2} \]

where: Tangential acceleration \( a_T = \frac{dv}{dt} \)

radial acceleration \( a_R = \frac{v^2}{R} \)

12. **WORK**

\[ W = \int_{S_1}^{S_2} \cos \theta \, ds = \left( m \frac{v_2^2}{2} - m \frac{v_1^2}{2} \right) + \left( mg \frac{y_2}{2} - mg \frac{y_1}{2} \right) + \int f \, ds \]

where: Kinetic Energy \( KE = \frac{mv^2}{2} \)

Gravitational Potential Energy \( PE = mg \)

For Elastic Potential Energy (e.g. Springs):

\[ W = \int kx \, dx = \frac{1}{2} kx^2 \]

13. **POWER** Instantaneous \( P = \frac{dW}{dt} = Fv \)

14. **MASS AND ENERGY**

\( KE = (m - m_0)c^2 \)

where \( m_0 = \) rest mass, \( c = \) velocity of light

15. **IMPULSE AND MOMENTUM**

An impulse force is a force that varies with time.

\[ J = \int F \, dt \]

Given the two bodies \( A \) \& \( B \), the impulse of a force acting on both bodies \((\int F \cdot dt)\) is equal to the change in momentum of each body \((\Delta P)\).

\[ dp = F \cdot dt \]

\[ P_f - P_i = \int_{t_1}^{t_2} F \cdot dt \]

Conservation of Momentum:

\[ m_A \cdot \frac{v_{A_1}}{A} + m_B \cdot \frac{v_{B_1}}{B} = m_A \cdot \frac{v_{A_2}}{A} + m_B \cdot \frac{v_{B_2}}{B} \]

The total momentum of a system can only be changed by external forces acting on the system.
16. **Rotation**

Instantaneous angular velocity: \( \omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \)

Instantaneous angular acceleration: \( \alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \)

Rotation with constant angular acceleration:

\[
\omega = \omega_0 + \alpha t \\
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \\
\omega^2 = \omega_0^2 + 2 \alpha \theta
\]

Relation between angular and linear velocity and acceleration:

\[
S = r \theta \\
v = r \omega \\
a_t = r \alpha \\
a_r = \frac{v^2}{r} = \frac{\omega^2}{r}
\]

Kinetic Energy of Rotation; Moment of Inertia; Torque(\( \tau \))

\[
I = \sum m_i r_i^2 \\
KE = \frac{1}{2} I \omega^2 \\
\tau = I \alpha
\]

**Angular Momentum**

\[
L = I \omega
\]

---

**Analogy Between Translation and Rotational Quantities**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Translation</th>
<th>Rotation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>( S )</td>
<td>( \Theta )</td>
<td>( S = r \Theta )</td>
</tr>
<tr>
<td>Velocity</td>
<td>( v = \frac{ds}{dt} )</td>
<td>( \omega = \frac{d\theta}{dt} )</td>
<td>( v = r \omega )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( \alpha = \frac{dv}{dt} )</td>
<td>( \alpha = \frac{d\omega}{dt} )</td>
<td>( a_r = r \alpha )</td>
</tr>
<tr>
<td>Resultant Force, Moment</td>
<td>( F )</td>
<td>( \tau )</td>
<td>( \tau = F_r )</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>( F = 0 )</td>
<td>( \tau = 0 )</td>
<td></td>
</tr>
<tr>
<td>Acceleration Constant</td>
<td>( v = v_0 + at )</td>
<td>( \omega = \omega_0 + \alpha t )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( s = v_0 t + \frac{1}{2} at^2 )</td>
<td>( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v^2 = v_0^2 + 2as )</td>
<td>( \omega^2 = \omega_0^2 + 2\alpha \theta )</td>
<td></td>
</tr>
<tr>
<td>Mass, Moment of Inertia</td>
<td>( m )</td>
<td>( I )</td>
<td>( I = \sum m_i r_i^2 )</td>
</tr>
<tr>
<td>Newton's Second Law</td>
<td>( F = ma )</td>
<td>( \tau = I \alpha )</td>
<td></td>
</tr>
<tr>
<td>Work</td>
<td>( W = \int Fds )</td>
<td>( W = \int \tau d\theta )</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>( P = Fv )</td>
<td>( P = \tau \omega )</td>
<td></td>
</tr>
<tr>
<td>Potential Energy</td>
<td>( PE = mgy )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kinetic Energy</td>
<td>( KE = \frac{1}{2} m v^2 )</td>
<td>( KE = \frac{1}{2} I \omega^2 )</td>
<td></td>
</tr>
<tr>
<td>Impulse</td>
<td>( \int Fdt )</td>
<td>( \int \tau dt )</td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>( m \nu )</td>
<td>( L = I \omega )</td>
<td></td>
</tr>
</tbody>
</table>
17. HARMONIC MOTION
Rectilinear Motion with constant acceleration

<table>
<thead>
<tr>
<th>Simple Harmonic Motion (in terms of ( \omega ) and ( x_0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ddot{x} = -\omega^2 x )</td>
</tr>
<tr>
<td>( \alpha = \omega^2 A \sin(\omega t + \theta_0) )</td>
</tr>
<tr>
<td>( v^2 = v_0^2 + 2a(x - x_0) )</td>
</tr>
<tr>
<td>( v = v_0 + at )</td>
</tr>
<tr>
<td>( x = x_0 + v_0 t + \frac{1}{2} at^2 )</td>
</tr>
<tr>
<td>( \omega = \frac{2\pi \sqrt{m}}{T} = \frac{2\pi \sqrt{k}}{m} )</td>
</tr>
<tr>
<td>( \sin \theta_0 = x_0^2 / A )</td>
</tr>
<tr>
<td>( \cos \theta_0 = v_0 / \omega A )</td>
</tr>
</tbody>
</table>

\( t \) = time from \( x_0 \) to any other \( x \)

Total Energy, \( E = KE + PE = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \) = constant

\( v_{max} = \omega A \) (at midpoint)
\( a_{max} = \omega^2 x_{max} \) (at ends)

Simple Pendulum: Restoring force = \( F = -mg \sin \theta = -mgx / L \) for small \( \theta \)

Period = \( T = 2\pi \sqrt{L / g} \)

Angular Harmonic Motion: Restoring torque = \( \tau = -k \theta \)

Period = \( T = 2\pi \sqrt{I / k} \)

18. HYDROSTATICS

Pascal's Law: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. \( P = F / A \)

Archimedes' Principle: Fluid exerts an upward buoyant force \( (F_y) \), on an immersed body, which is equal to the weight of the fluid displaced and whose line of action passes through the center of gravity of the displaced fluid.

Specific Gravity of a body = \( \frac{\text{density of body}}{\text{density of water}} = \frac{\text{weight of body}}{\text{wt. of eq. volume of water}} \)
19. HYDRODYNAMICS AND VISCOSITY

Bernoulli's Equation, applicable to streamline flow without resistance:

Net work done on a system \( P_i - P_2 = \frac{1}{g} (v_1^2 - v_2^2) + y_2 - y_1 \)

where \( P \) is absolute pressure

**Venturi:** \( P_i - P_2 = gh \)

![Venturi Diagram]

**Speed of Efflux:** \( v_2 = \frac{v_1 A_1}{A_2} \)

![Speed of Efflux Diagram]

If \( A_1 \gg A_2 \), \( v_2 \approx 2gh \) and \( v_1^2 \) is much less than \( v_2^2 \), and the speed acquired is that of any falling body through height \( (h) \).

For enclosed vessel with pressure: \( v_2 = \frac{\sqrt{2(P - P_a)/\rho}}{2gh} + \sqrt{2gh} \)

![Enclosed Vessel Diagram]

**Viscosity:** Drag Force = \( F = \frac{1}{2} \rho AVR \) where \( \nu \) = coeff. of viscosity

**Reynolds Number:** A combination of four factors which determine whether the flow of a viscous fluid through a pipe is laminar or turbulent.

\[ N_R = \frac{\rho V D}{\mu} \]

\( N_R \) (0-2000) = laminar, \( N_R \) (\( \geq 3000 \)) = turbulent

Effect of friction is to introduce an additional pressure head to

Bernoulli's equation \( \frac{P_1 - P_2}{\rho g} = f \frac{1}{2} \frac{V^2}{2g} + (y_2 - y_1) \)

where \( f \) = friction factor = \( \frac{6\mu}{\rho} \)

for laminar flow and varies as a fractional power of \( N_R \) for turbulent flow.
20. TEMPERATURE-EXPANSION

Fahrenheit: \( T_F = \frac{9}{5} T_C + 32 \)  
Centigrade: \( T_C = \frac{5}{9} (T_F - 32) \)

Absolute Zero: \(-460^\circ F = -273^\circ C\)

Coeff. of Linear Expansion: \( \alpha = \frac{\Delta L}{L_0 \Delta T} \)

Surface and Volume Expansion: Surface: \( \Delta A = 2 \alpha A_0 \Delta T \)  
Volume: \( \Delta V = 3 \alpha V_0 \Delta T \)

21. QUANTITY OF HEAT

778 ft-lb of mechanical energy, when converted to heat, will raise temperature of 1 lb. of water through 1 °F.

778 ft-lb = 1 BTU  
4.186 joules = 1 gm-cal

Heat Capacity = ratio of heat supplied to corresponding temperature rise = \( C \)

Specific Heat Capacity = \( c_p = \frac{\text{Heat Capacity}}{\text{mass}} = \frac{Q}{m \Delta t} \)  
\( Q = mc_f \Delta t \)

Heat of Transformation (fusion or vaporization) = \( Q = mL \)
where \( L \) represents heat absorbed or liberated in change of phase.

22. TRANSFER OF HEAT

Conduction: Quantity of heat transmitted per second from one slab face to the opposite face = \( H = kA \Delta T \), where \( k \) is coeff. of thermal conductivity.

Convection: Transfer of heat from one place to another by actual motion of hot material.

Heat Convection Current = \( H = hA \Delta T \), where \( h \) is convection coeff.

Radiation: Ability of substance to emit radiation when heated and is proportional to its ability to absorb radiation, e.g., black body absorbs all.

Radiant Energy Rate of Emission = \( R = e \sigma T^4 \) ergs/sec-cm\(^2\) (absolute \( T \))
where \( e \) = emissivity of surface, varies from 0 to 1.
\( \sigma = 5.67 \times 10^{-8} \) ergs per sec per cm\(^2\) per degree Kelvin.

23. THERMAL PROPERTIES OF SOLIDS, LIQUIDS, GASES

Boyle's Law: Initial \( p_1v_1 = \) final \( p_2v_2 \) for a given temperature.

\[ \rho \]

\[ t_1, t_3, t_4, v \]

Equation of State of an Ideal Gas: \( pV = nRT \)
where \( n \) = number of moles  
= mass of the gas/molecular weight
\( R \) = Universal gas constant = 1.99 cal/mole-°C
\( = C_p - C_v \) (molecular heat capacity)
Mole = gram molecular weight = molecular weight of substance expressed in grams. Thus 1 mole of oxygen is 32 g of oxygen; one mole of an ideal gas occupies 22.4 liters at 273°K at 1 atm.

**Ideal Gas Law:** \[ p_1 V_1 T_1 = p_2 V_2 T_2 \] (absolute \( T \))

**Kinetic Theory of an Ideal Gas:** \( pV = N k T \)
where \( N \) = total number of molecules in container of Volume (V).
\( k \) = Boltzmann's constant = \( R/N_0 \) \( (N_0 = N/n) \)

\( N_0 \) = Avogadro's Number = \( 6.02 \times 10^{23} \) molecules/mole

**Absolute Humidity:** mass of water vapor per unit volume

**Dalton's Law:** The partial pressure of each of the component gases of a gas mixture is very nearly the same as would be the actual pressure of that component alone if it occupied the same volume as does the mixture, i.e., each of the gases of a gas mixture behaves independently of the others.

**Relative Humidity (\%) = 100 \times \frac{\text{partial pressure of water vapor}}{\text{vapor pressure at same temperature}}.**

**24. FIRST LAW OF THERMODYNAMICS: "Law of Energy"**
One can obtain no more than 778 ft-lb of mechanical work from every BTU of heat. The First Law denies the possibility of creating or destroying energy.

Work in changing the Volume = \( W = \int_{V_1}^{V_f} p \, dV \)

where \( V_1 \) = initial volume \( V_f \) = final volume \( p \) = pressure exerted upon the volume, not necessarily being constant \( dV \) = differential change in the volume

**Accepted Convention of Work:** Work done by a substance is positive.
Work done on a substance is negative.

**First Law of Thermo:** \( Q - W = \Delta U \), where \( U \) = internal energy

To apply Law: (1) All quantities must be expressed in same units
(2) \( Q \) is positive when heat goes into system.
(3) \( W \) is positive when work goes out of system.

**Adiabatic Process:** A process that takes place in such a manner that no heat enters or leaves a system, i.e. \( -W = U_2 - U_1 \)

**Isovolumeric Process:** Process which volume remains unchanged \( Q = U_2 - U_1 \)

**Isobaric Process:** Process at constant pressure
\( Q = mL + (U_{\text{vapor}} - U_{\text{liquid}}) + p(V_{\text{vapor}} - V_{\text{liquid}}) \)

where \( L \) = heat of vaporization
25. **SECOND LAW OF THERMODYNAMICS**  
"Law of Entropy"

The second law goes beyond the first and states that 100% conversion of heat into mechanical energy is not possible by any form of engine.

**Second Law:** It is impossible to construct an engine that, operating in a cycle, will produce no effect other than the extraction of heat from a source and the conversion of this heat completely into work.

Thermal Efficiency = $E = \text{Work Output/Heat Input} = \frac{W}{Q_2}$

26. **INTERNAL COMBUSTION ENGINES**

**Otto Cycle**

$ab = \text{adiabatic}$  
$cd = \text{adiabatic}$  
compression ratio $\approx 7$  
efficiency $\approx 54\%$

**Diesel Engine:**

$ab = \text{adiabatic}$  
$cd = \text{adiabatic}$  
compression ratio $\approx \frac{v_1}{v_2} \approx 15$  
efficiency $\approx 56\%$

**Steam Engine:**

$ab = \text{adiabatic}$  
$cd = \text{adiabatic}$  
efficiency $\approx 32\%$

**Refrigerator:** Heat engine operating in reverse

$Q_2 = W + Q_1$

$Q_2 = Q_1 + W$ where $Q_2$ = heat delivered to external cooling coils

Coefficient of performance $Q_1/(Q_2 - Q_1) = 2$ to 6
Carnot Cycle: Bounded by two isothermals and two adiabatics. Thus all heat input is supplied at a single high temperature, and all heat output is rejected at a single lower temperature. No engine operating between two given temperatures can be more efficient than a Carnot Engine operating between the same two temperatures. All Carnot engines operating between the same temperatures have the same efficiency, irrespective of the nature of the working substance.

The efficiency of the cycle can be determined by using the above T-S diagram. The heat transfer during the isothermal processes is, by definition \( q = -\int T \, ds \). The work done in a cycle is equal to the heat received from the high temperature source minus the heat rejected from the lower temperature sink (\( W_{\text{net}} = Q_S - Q_R \)). The thermal efficiency in a heat engine is:

\[
\eta_T = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{Q_S - Q_R}{Q_R} = \frac{(T_A - T_B)(S_3 - S_2)}{T_A(S_3 - S_2)} = 1 - \frac{T_B}{T_A}
\]

where: the temperatures must be evaluated on the Kelvin scale.

Entropy: The entropy change in any isothermal reversible process, equals the heat added divided by the absolute temperature.

\[
S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T} \quad \text{(along a reversible path)}
\]

In an adiabatic process no heat is allowed to enter or leave the system. Hence, \( Q = 0 \) and there is no \( \Delta S \). Therefore, every reversible adiabatic process is one of constant entropy (isentropic).

Principle of Entropy Increase: When all systems taking part in a process are included, the entropy either remains constant or increases, i.e., no process is possible in which entropy decreases.

Significance of Entropy: Entropy represents the quality or usability of a given amount of energy. It represents the extent to which the Universe "runs down" in every natural process. When entropy increases, energy becomes more unavailable (irreversible).
27. WAVE MOTION

Periodic Waves: Transverse when motion of particles are at right angles to direction of travel; Longitudinal if particles move in direction of propagation.

Velocity of Propagation = \( c = f \lambda \), where \( f \) = frequency, \( \lambda \) = wavelength

Transverse Pulse Speed = \( c = \sqrt{\frac{s}{n}} \), where \( s \) = tension, \( n \) = linear density

Longitudinal Pulse Speed = \( c = \sqrt{\frac{Y}{\rho}} \), for solid

where \( Y \) = Young's modulus of elasticity, \( \rho \) = density of material

\( c = \sqrt{\frac{B}{\rho}} \), for liquid or gas

where \( B \) = Bulk modulus

Acoustical Phenomena:

Sound is longitudinal wave motion that can be perceived by the auditory nerve. It consists of a series of condensations and rarefactions, and can be transmitted by any type of elastic matter, but not by vacuum. Sound waves range from 20 to 20,000 vibrations per second.

Intensity = the time average rate at which energy is transported by the wave per unit area = \( I = P^2/2c \)

where \( P \) = pressure amplitude, \( c \) = velocity of sound wave

Intensity Level \( \beta = 10 \log I/I_0 \) (decibels) where \( I_0 \) = ref. intensity

Doppler Effect: When a source of sound waves and the ear are approaching each other, the pitch of the sound is apparently increased, while if they are receding from each other, the pitch is apparently lowered. The pitch of a sound as heard depends on the number of waves reaching the ear per second, (or perceived frequency). Doppler Effect is not confined to sound waves but applies also to light and other electro-magnetic waves such as radar.

28. ELECTRICAL

Ionization: Process of losing or gaining electrons.

Coulomb's Law: The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. \( F = k q_1 q_2/r^2 \), where \( k \) = dielectric constant (1 for vacuum). \( k = 1 \) defines a unit charge of 1 Statcoulomb which is the charge that would repel one equal charge of the same sign with a force of one dyne when the charges are separated by 1 cm.

1 Coulomb = quantity of charge which in one second crosses a section of a conductor in which there is a constant current of 1 amp.

1 coulomb = \( 3 \times 10^9 \) statcoulombs.
Capacitance: of a capacitor is the ratio of the charge \( Q \) on either conductor to the potential difference between the conductors

\[
C = \frac{Q}{V_{ab}} = 1 \text{ farad if 1 coulomb is transferred per volt}
\]

Series: \( \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \)  
Parallel: \( C = C_1 + C_2 + C_3 \)

By Loop Law: \( \mathbf{E} - iR - \frac{q}{C} = 0 \), where \( i = \frac{dq}{dt} \)

Solving the differential equation: \( q = C \mathbf{E}(1 - e^{-t/RC}) \)

\[
i = \frac{dq}{dt} = \frac{\mathbf{E}}{R}(e^{-t/RC}) \text{ for a charging capacitor.}
\]

Potential: at any point of an electrostatic field is defined as the potential energy per unit charge. \( V = E_p/q' = k\mathbf{E}/q \)

Electron Volt: Change in potential energy of a particle having charge \( q \) when it moves from \( V_a \) to \( V_b \). 1 ev = \( 1.6 \times 10^{-19} \text{ J} \).

Ohm's Law: Resistance of a conductor is 1 ohm if the potential difference between the terminals of the conductor is 1 volt when the current in the conductor is 1 amp.

\[
R = \frac{V}{I} \quad \text{Series: } R = R_1 + R_2 + R_3 \quad \text{Parallel: } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

Kirchoff's Law:
Point Law-The algebraic sum of the currents to and from any branch point of a network is zero: \( \sum i = 0 \)

Loop Law: The algebraic sum of the emf's in any loop of a network of the IR products in the same loop. \( \sum V = \sum IR \)

\[
\text{Power} = IV = I^2R = V^2/R \quad \text{watt} = 10^7 \text{ ergs/sec} = 1 \text{ joule/sec} = \text{power from 1 amp by 1 volt}
\]

Energy: \( Pt = IVt = I^2Rt = V^2t/R = \text{watt-sec or joules} \)

29. ELECTROLYSIS

Faraday's Law: The number of gram-equivalent weights of a substance deposited, liberated, dissolved, or reacted at an electrode is equal to the number of faradays of electricity transferred through the electrolyte.

1 faraday = 96,500 coulombs

The mass of different substances liberated by the same quantity of electricity are proportional to their equivalent weights.

chemical equivalent=atomic weight divided by valence. When chemical equivalent is expressed in grams, it is gram-equivalent.

Lead Storage Battery: \( \text{PbO}_2 + \text{SO}_4^{2-} + 4\text{H}^+ + 2e^- \rightarrow \text{PbSO}_4 + 2\text{H}_2\text{O} \)
30. **MAGNETIC FIELD**: is said to exist at a point if a force (over and above any electrostatic force) is exerted on a moving charge at any point. When the velocity of the moving charge is perpendicular to the magnetic field the force is perpendicular to both the magnetic field and the velocity. (left hand rule)

\[ F = F \cdot v \sin \phi \]

One WEBER per square meter is the magnetic field \((B)\) in which one coulomb of charge, moving with a component of velocity perpendicular to the field \((v \sin \phi)\) equal to one meter per second, is acted on by a force of one newton.

\[ B = \frac{F}{q \cdot v \sin \phi} \quad \text{or} \quad F = q \cdot v \cdot B \sin \phi \]

**Force on wire in magnetic field**

\[ F = F \cdot (\text{dynamometer}) = I \cdot l \times B \quad (I = \text{length of wire}) \]

Forces on the sides of a current-carrying loop in a magnetic field. Resultant of the forces is a couple of moment

\[ \tau = I A \times B \]

where \(A\) = vector area of loop

\(m = I A\) = magnetic moment

\(\tau\) = torque vector, is in direction of the vector product \(A \times B\) and points along the +y axis.

**Magnetic Flux** = total number of lines of induction threading through a surface

\[ \Phi = \int B \cdot n \, dA \]
Faraday's Law (Magnetics): The induced emf in the current is numerically equal to the rate of charge of the flux through it.

\[ E = -\frac{d\Phi}{dt} = B \cdot v \cdot l \]

(right hand rule)

Lenz' Law: The direction of an induced current is such as to oppose the cause producing it. If a conductor has \( N \) turns and each turn cuts \( \Phi \) lines in \( t \) seconds, average induced emf \( E \) (volts) = \( -N \frac{\Phi}{t} \times 10^{-8} \)

31. **INDUCTANCE**: an emf is induced in a stationary circuit whenever the magnetic flux through the circuit varies with time.

Self-Inductance \( L = N \frac{\Phi}{I} \) = weber-turn per amp = Henry

Self-Induced EMF \( E = -L \frac{di}{dt} \)

Energy Supplied To Inductor = \( W = L \int_0^t di = LI^2/2 \)

By Loop Law: \( \epsilon - iR - L\frac{di}{dt} = 0 \)

Solving the Differential Equation: \( i = (\frac{\epsilon}{R})[1 - e^{(-Rt/L)}] \) in magnetizing inductor.

32. **ALTERNATING CURRENTS**

Sinusoidal Alternating Potential Difference = \( v = V \cos \omega t \)

ELI the ICE man (phase hint):

Inductive circuit: \( E L I, E \) leads \( I \)

Capacitive circuit: \( I C E, I \) leads \( E \)

Resistance circuit: current and voltage in phase
Inductive Reactance \( X_L = \omega L \)  
Capacitive Reactance \( X_C = 1/\omega C \)

R-L-C-Series Circuit: \( V_R = IR, \) \( V_L = IX_L, \) \( V_C = IX_C \)

Impedance \( Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + [L - (1/C)]^2} \) (ohms)

Natural Frequency of L-C Circuit \( \omega = \sqrt{1/\omega \text{C}} \) (resonance)

\( \omega \) = angular frequency of electrical oscillations
Amount of resistance in R-L-C circuit determines damping effect.

The Current in a RLC circuit is:
\[
i = \frac{\mathcal{E}_{\text{max}} \sin(\omega t + \phi)}{R}
\]

where: \( \phi = \tan^{-1}(X_C - X_L/R) \)

\( \mathcal{E}_{\text{max}} \) = maximum voltage generated by power source.

Root Mean Square Value = the square root of the average value of the square of the current or voltage (ac meters calibrated to read this)

\[
I_{\text{rms}} = \sqrt{\left(\frac{\mathcal{E}}{R}\right)^2} = \mathcal{I}_{\text{rms}}
\]

\( V_{\text{rms}} = \mathcal{V}_{\text{rms}} \)

Power \( = I_{\text{rms}}^2 R = V_{\text{rms}} I_{\text{rms}} \cos \phi \), where \( \cos \phi \) = power factor

Low Power Factor (large angle of lag or lead) is usually undesirable in power circuits because, for given potential difference, large current is needed to supply given amount of power, with corresponding large heat losses in transmission lines.

Transformer: For efficiency, it is desirable to transmit power at high voltage and small current, reducing \( I^2 R \) heat losses. Considerations of safety and insulation require low voltages in equipment.

\( V_2/V_1 = N_2/N_1 \) (turns ratio) If \( V_2 \) greater than \( V_1 \), this leads to step up, If \( V_2 \) is less than \( V_1 \) this leads to step down.
33. ELECTROMAGNETIC WAVES

Electromagnetic Field: A region in which both electric and magnetic fields exist.

Electromagnetic Wave: Moving pattern of electric and magnetic fields.

Wave Features:
1. Wave direction of propagation is transverse to electric and magnetic fields.
2. Electric and magnetic intensities are perpendicular to each other at every point, as well as being perpendicular to the direction of travel.
3. The two fields are in phase with each other; the magnetic field is maximum where potential difference is maximum.
4. Waves travel in vacuum with definite and unchanging speed.

Speed of E-M Wave = \( C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \)
where \( \mu_0 \) and \( \varepsilon_0 \) are, respectively, permeability and permittivity of the medium. In free space \( C = 3 \times 10^8 \) m/sec = 1 x.

34. LIGHT PROPAGATION

The phenomena of light propagation may best be explained by the E-M wave theory, while the interaction of light with matter, in the process of emission and absorption, is a corpuscular phenomena.

Law of Reflection: The incident and reflected rays and the normal all lie in the same plane; the angle of reflection (\( \phi_r \)) is equal to the angle of incidence (\( \phi_i \)) for all colors and any pair of substances.

\[ \phi_i = \phi_r \]

Law of Refraction: The incident and refracted rays and the normal to the surface all lie in the same plane; and \( \sin \phi_i / \sin \phi_b = \) constant.

This is SNELL's LAW, \( n_a \sin \phi_i = n_b \sin \phi_b \)

It is evident from Snell's Law that the angle of refraction is always less than the angle of incidence for a ray passing from a medium of smaller into one of larger index, as from air into glass. In such a case the ray is bent toward the normal. For larger to smaller indices, the reverse is true (i.e., the ray is bent away from the normal).
Index of Refraction: \( n = \frac{\sin \phi_v}{\sin \phi_a} \) (The constant in Snell's Law)

where \( \phi_v \) is angle of incidence for monochromatic light in vacuum.

\( \phi_a \) is angle of refraction in the substance.

Dispersion: With few exceptions, velocity of light in a material substance is less than velocity in free space. Further more, while light of all wavelengths travels with same velocity in empty space, the velocity in material substances is different for different wavelengths. This effect is called Dispersion. On emerging from a prism, light is spread out into a fan-shaped beam. The light is said to be dispersed into a spectrum.

35. MIRRORS AND LENSES

Reflection at a Plane Mirror:

\[ m = \text{magnification} = \frac{y'}{y} = -\frac{S'}{S} = +1 \]

Convention: (1) If direction from object to reflecting or refracting surface is same as that of light incident on the surface, the object distance is positive.

(2) If direction from the reflecting or refracting surface to the image point is the same as that of the light reflected or refracted from the surface, the image distance \((S')\) is positive.

Reflection at a Spherical Mirror:

\[ m = \frac{y'}{y} = -\frac{S'}{S} \]

\[ \frac{1}{S} + \frac{1}{S'} = \frac{2}{R} = \frac{1}{f} \]

Convention: If the direction from a reflecting or refracting surface to the center of curvature is the same as that of the reflected or refracted light, the radius of curvature is positive.

Focal Length = \( f \)= distance between the vertex of a mirror and the focal point. \( f = \frac{R}{2} \) for a convex and concave mirror.
Focal Point: considered as the image point of an infinitely distant object point on the mirror axis, or the object point of an infinitely distant point.

\[ S = \infty \]
\[ F = \frac{R}{2} = S' \]

Thin Lens: Lenamaker's Equation:

\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{S} + \frac{1}{S'} \]

magnification = \( m = -\frac{S'}{S} \)

Graphical method:

1. a ray parallel to the axis, after refraction by the lens, passes through the second focal point of a converging lens, or appears to come from the second focal point of a diverging lens.

2. a ray through the center of the lens is not appreciatively deviated, since the two lens surfaces through which the central ray passes are very nearly parallel if the lens is thin.

3. a ray through (or proceeding toward) the first focal point emerges parallel to the axis.

Interference: refers to any situation in which two or more waves overlap in space.

Principle of Linear Superposition: When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

Diffraction: refers to the spreading of light waves around the edges of apertures and opaque objects, and to the interference patterns produced by this deviation of light from a rectilinear path. The distribution of light and dark on any screen after a beam of light has been partially blocked by a perforated diagram is called a DIFFRACTION PATTERN.
Diffraction Grating: A large number of parallel slits, all of the same width and spaced at regular intervals
Diffraction grating equation: \( \sin \theta = m \frac{\lambda}{d} \), \( m=1,2,3, \) etc.
If the slit is illuminated with white light, a continuous group of images is formed side by side, i.e., white light dispersed into continuous spectra. In contrast to single spectrum produced by a prism, a grating forms a number of spectra on either side of normal. Grating widely used in spectrometry to complete \( \lambda \).

36. ATOMIC PHYSICS

Photon: a small bundle of energy conveyed by an electromagnetic wave, such as light, whose magnitude is proportional to the wave's frequency.

\[ E = hf \], where \( h = \text{Planck's constant} = 6.63 \times 10^{-34} \text{ Js} \).

Bohr Atom:

1\textsuperscript{st} postulate - an electron in an atom can revolve in certain stable orbits without the emission of radiant energy.

2\textsuperscript{nd} postulate - an electron may make a transition from one of its specific non-radiating orbits to another of lower energy. When it does so, a single photon is emitted, having energy equal to the energy difference between the initial and final states, and with frequency \( f \).

\[ hf = E_{\text{final}} - E_{\text{initial}} \]

Wave Mechanics:
DeBroglie in 1923 said that since light is dualistic in nature, behaving in some aspects like particles, the same may be true of matter, i.e., electrons and protons might in some circumstances behave like waves. This Wave hypothesis was developed by Heisenberg, Schrodinger and others into a complete theory called WAVE MECHANICS or QUANTUM MECHANICS. Electrons in atoms are visualized as diffuse clouds surrounding the nucleus. This theory assigns definite energy states to an atom. In the hydrogen atom the energies and angular momentum are the same as Bohr's Theory.
Principles of Quantum Mechanics lead to a Wave Equation (Schrodinger's) that must be satisfied by an electron in an atom, subject to boundary conditions. Considering an electron as a wave extending in a circle around the nucleus, the circumference must include some integer number of wavelengths. According to Wave Mechanics, the wavelength \( \lambda = \frac{h}{mv} \) and angular momentum of the electron \( mL = nh/2\pi \), where \( n=1,2,3,\ldots \) It is possible to make a list of the possible sets of quantum numbers and thus of the possible states of electrons in an atom. The quantum-mechanical state of an electron is identified not by a single quantum number \( n \) as in the Bohr model of the hydrogen atom, but by a set of four quantum numbers, all integers:

1. \( n \) = principle quantum #, any positive integer. Energy of the state and its distance from the nucleus increase with \( n \).

2. \( l \) = magnitude of angular momentum. Value can be zero or any positive # up to and including \( n - 1 \).

3. \( m \) = component of angular momentum in a particular axis direction. The value for \( m \) can be zero or any positive or negative integer up to and including \( \pm 1 \).

4. \( s \) = spin quantum # having one of two values \( \pm 1 \).

Pauli Exclusion Principle: In an atom, no two electrons can have all four quantum numbers the same.

Nuclear Structure: Every atom can be expressed with the aid of 3 numbers-

1. Atomic Number \( Z \) = number of protons in nucleus, (also the number of orbital electrons for an electrically neutral atom.)

2. Mass Number \( A \) = number of nucleons (protons and neutrons) in nucleus.

3. Neutrons \( N \) = number of neutrons in nucleus.

\[ A = Z + N \] e.g., \( ^4_2\text{He} \) (alpha particle) where \( Z=2, A=4 \)

Isotopes: Nuclei of a given element having different mass numbers; has same # of protons but different # of neutrons.

Radioactive Transformation: The number of radioactive nuclei decreases continuously as some nuclei disintegrate.

The rate of disintegration = \( \frac{dN}{dt} = -\lambda N \), where \( N \) = # of radioactive nuclei in a sample at time \( t \). If \( N_0 \) = # at some time \( t=0 \), then the number \( N \) remaining at a later time \( t \) is: \( N = N_0 e^{-\lambda t} \)

Half-Life: The time at which the # of radioactive nuclei has decreased to one-half the # at \( t=0 \). \( t_h = (1/\lambda) \ln 2 \)

Activity = # of disintegrations per unit time.

Curie = \( 3.7 \times 10^{10} \) decays per second
Nuclear Binding Energy: is the energy that would have to be supplied to break up a nucleus into the component neutrons and protons (plus their associated electrons. Viewed as supplying energy to remove the nucleons from a low energy well.)

Equivalently, it is also the energy that would be released if a nucleus could be made from it's component parts. (Viewed as releasing energy as the components "fall" into a low energy well.

Nuclear Fission: When uranium (Z=92) is bombarded with neutrons, the uranium nucleus is said to undergo fission where an enormous amount of energy, 200 MeV/fission, is released when uranium splits up. Since the rest mass of a uranium atom exceeds the sum of the rest masses of the fission products, it follows from the Einstein mass energy relation that the extra energy released during fission is transformed into kinetic energy of the fission fragments. As these fragments move through the fuel clod, moderator, etc they generate heat. Uranium fission is accomplished by either fast or slow neutrons. Of the two most abundant isotopes of uranium, $^{235}\text{U}$ and $^{238}\text{U}$, both may be split by a fast neutron, whereas only one, $^{235}\text{U}$, is split by a slow neutron. In the fission of $^{235}\text{U}$, an average of 2.5 neutrons per fission is produced. They are fast neutrons and are almost all released at the instant of fission. In a thermal reactor they must first be moderated to be useful. A fast neutron has approximately 5 MeV of energy, whereas a thermal neutron has approximately 0.025 eV of energy. For efficiency in a reactor using thermal neutrons for fission, a "slowing down process" or "moderation" is required. A Chain Reaction is a self-sustaining series of events, which once started, will continue. For a uranium chain reaction, a neutron causes one uranium atom to undergo fission, during which a large amount of energy and several neutrons are emitted. These neutrons then cause fission in neighboring Uranium nuclei which also release energy and more neutrons. If the chain reaction proceeds in a controlled manner, the device is a nuclear reactor; if the chain reaction is fast and uncontrolled, the device is an atomic bomb.

Nuclear Fusion: Involves the combination of two light nuclei (e.g. Hydrogen) to form a nucleus which is more complex but whose rest mass is less than the sum of the rest masses of the original nuclei (same as for fission). Examples of such energy-liberating reactions are:

$$\begin{align*}
1^1\text{H} + 1^1\text{H} &\rightarrow 1^4\text{H} + 1^0\text{e} \\
1^1\text{H}^2 + 1^1\text{H} &\rightarrow 2^4\text{He} + \text{radiation} \\
2^1\text{H}^3 + 2^4\text{He}^3 &\rightarrow 2^4\text{He}^4 + 1^1\text{H} + 1^1\text{H}
\end{align*}$$

These reactions now as proton-proton chain, are believed to take place in the interior of the sun and stars. Temperatures and pressures similar to those in the stars may be achieved on earth at the moment of explosion of a uranium or plutonium fission bomb. If the fission bomb is surrounded by proper proportions of hydrogen isotopes, these may be caused to combine into helium and liberate still more energy (hydrogen bomb).
MATHEMATICS QUESTIONS

I. GENERAL QUESTIONS

1. What is a solution to this equation: \((1 - y)^2 + 2xy = 0\)
   A. \((1,1)\)
   B. \((1,1)\)
   C. \((1,1)\)
   D. \((1,0)\)

2. The locus of \(p(x,y)\), such that the difference of their distance from two fixed points is constant, \(a(n)\):
   A. ellipse
   B. hyperbola
   C. parabola
   D. circle

3. A propeller-driven plane and a jet travel 3,000 miles. The velocity of the prop plane is \(1/3\) the velocity of the jet. It takes the prop plane 10 hours longer to complete the trip. What is the velocity of the jet?

4. What is the center of \(x^2 + y^2 - 2x - 4y - 17 = 0\) ?

5. Simplify: \(\frac{y^2 - 1}{(y + 1)^2} \cdot \frac{1}{y - 1} \cdot \frac{y + 1}{1}\)

6. Simplify: \(\frac{a^4 + b^4}{a^2 + b^2}\)

7. Simplify: \(\frac{3 + 21}{3 - 21}\)

8. Give a basic explanation of a Fourier series, Taylor series, and simple Laplace transformations.

9. What is 9435 in base 8?

10. What is 4361(base 8) in base 10?

11. What is Bayes Theorem?

12. Solve a system of 3 simultaneous linear equations in three variables:
   \[\begin{align*}
x + 4y + 5z &= 2 \\
2x + y + 3z &= 1 \\
3x + y - z &= 3
\end{align*}\]
13. Given the series below, find the series function in terms of \( n \):

\[ S = 2, 6, 12, 20, 30, 42, \ldots \]

14. Show how to find the roots of a fourth or fifth degree curve involving techniques of numerical analysis.

15. What is a logarithm?

16. The number of square feet in a circle is equal to the number in feet of the circle's circumference. What is the circle's radius?

17. Derive the equation of a circle around any point.

18. Given a closed box where the length is twice the height, the width is 10 meters less than the length, and the total surface area is ten times the width times the height, what are the dimensions?

19. How is \( e \), the base of natural logarithms, defined?

20. Derive the Quadratic Equation.

21. What are the cross products and dot products for two vectors?

22. What geometric surface encloses maximum volume with minimum surface area? How would you prove it?

II. GRAPHING

1. Graph: \( x^2 + 2x - 3 - y = 0 \)

2. What type of smooth curve would go through these three points? What would its equation be?

3. Graph: \( y = e^{-x^2} \)

4. Graph: \( y = e^{-2} \)

5. Graph: \( y = e^{-x} \)

6. Graph: \( y = x^3 - x^2 \)
7. Graph: \( y = e^{-x^3} \)

8. Graph: \( y = 12x^4 - 2x^3 + 8 \)

9. Graph \( x^2 + x - 6 = y \) and from \( x = -4 \) to \( x = 3 \) find the minimum value of the function.

10. Graph: \( x = 2(y - 2)^2 \)

11. Graph: \( y = x^x \)

12. Graph: \( f(x) = x^2e^{-x^2} \)

III. DIFFERENTIAL AND INTEGRAL CALCULUS

1. What is a differential? What is a derivative? What is the significance of the first derivative? the second derivative?

2. Show the ability to integrate or differentiate by the following methods: parts, chain rule, division rule \((f(x)/g(x))\).

3. What is the difference between a definite and an indefinite integral?

4. How is an integral related to a graph of a function?

5. Define integration.

6. Find the derivative of \( y = 2x + x^2e^{-x} \).

7. Differentiate the following:
   a. \( xsin^2x \)
   b. \( xe^x + (x^3/sin(x)) - sin^2(x) + (x^2 + 1)^3 \)
   c. \( e^x \)
   d. \( sin(x) \)
   e. \( cos(x) \)
   f. \( ln(x) \)
   g. \( cot(x) \)
   h. \( tan(x) \)
   i. \( sec(x) \)
   j. \( csc(x) \)
   k. \( 10^x \)
   l. \( y^5 + (cos y)(e^y) + sin(y^2/3) \)
   m. \( x + x^3 \cdot sin(x)cos(x) + sin(x) \)
   n. \( y^2e^y \)
8. Given the figure below, determine the value of $x$ so that when the corners are removed, the five-sided box formed will have maximum volume.

9. Find the point of inflection of the following functions:
   a. $y = \sin(x)$
   b. $y = x^4 - 2x^2$

10. When do you use l'Hôpital's rule?

11. What is $\lim_{x \to 0} \frac{\sin x}{x}$

12. Find the maximum or minimum of a parabola and determine if it is a maximum or minimum.

13. Given a 100 ft. long house and 80 feet of fence, determine the shape of the fence which results in the largest fenced-in area between the house and the fence.

14. Prove that the derivative of $x^2$ is $2x$.

15. Analyze the curve $y = 1 + e^{-x}$ by finding the first two derivatives, maxima, minima, and inflection points.

16. How would you explain integration to an incoming freshman?

17. Integrate the following functions:
   a. $\sec^2x \, dx$
   b. $(2x + 1) \, dx$
   c. $(y + 3)(y + 1) \, dy$
   d. $x\sin(x) \, dx$
   e. $d(\sin^2(x) \cos(x)) / dx$
   f. $x^2 \, e^x - x^2 \, dx$
   g. $x^2 + xe^{-x}$
   h. $e^{-x} \cdot xe^{-x}$
   i. $(x + 2) \, d(x + 1)$
   j. $xe^{-x} \, dx$
   k. $R / 2 / 2$
   l. $\sin d \, d \, dr$
   m. $\sec(u) \tan(u) \, du$
   n. $\sec^2(u) \, du$
   o. $\csc^2(u) \, du$
   p. $\sec x \, du$
   q. $\cos(x) \, du$
   r. $\sin(x) \, du$
   s. $\sec(x) \tan(x) \, du$
   t. $2x + 1 \, du$
s. \( \frac{dx}{x} \)

18. Find the area of a circle by integration.

19. Find the area of a triangle by integration.

20. Find the area under the curve \( y = 4 - x^2 \) above the x-axis.

21. Find the volume of a cone of radius \( R \) and height \( H \) using integration. What incremental volume would you use? What are the dimensions of the incremental volume?

22. Derive the formula for the volume of a pyramid, \( V = \text{(Area of base)}(H) \) as depicted below:

23. Derive an equation for the surface area of a cylinder (including the ends).

24. Find the area bounded by the functions \( y = 4 - x^2 \), \( y = x^2 - 1 \), and the y-axis in the first quadrant.

25. Given the figure below with uniform mass, what is the y-coordinate of the center of gravity?

26. Find the area between the points \( x=0 \) and \( x=6 \) but below the x-axis for the graph of \( x^2 + x - 6 = y \).

27. Find areas between the two curves \( y = x^3 \) and \( y = x^2 \).

28. Determine the area between \( y = x^2 \) and \( y = x \) using integral calculus.

29. Rotate \( y = 1/x \) about the x-axis and find volume from 1 to infinity.
30. Determine the area between two concentric circles of radii 1 and 2 respectively using calculus.

IV. DIFFERENTIAL EQUATIONS

1. Solve \( \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = e^{-t} \)

2. What is the Laplace transform of \( f(t) = t \)?

3. Solve \( \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = \sin(x) \)

4. Solve \( \frac{dx}{dt} = \frac{x}{k} \)

5. Solve a general and specific homogeneous equation with derivatives:
   \( \frac{dy}{dx} + Ky = 10 \)

6. Explain how to solve the following differential equation:
   \( \frac{d^2A}{dt^2} + \frac{dA}{dt} + A = 0 \)

7. Solve \( y - 3y' = 0 \) for \( y(0) = 3 \).

V. PROBABILITY AND STATISTICS

1. With three men and seven seats, what is the number of consecutive three seats possible?

2. What is the difference between the mean, median, and mode in statistics?

3. What is a Gaussian distribution?

4. There are five persons in a room. Assuming that each person shakes hands with no one person more than once, how many hand shakes are there?

5. There is a box full of 50 black balls and 50 red balls.
   a. What is the probability of getting a red ball on the second pick if the first ball picked is red (without replacement)?
   b. What is the probability of getting a black ball on the second pick if the first ball picked is red (without replacement)?
PHYSICS PROBLEMS

I. WORK ENERGY & MOMENTUM

1. A block of mass \( M \), is attached to a string which is attached to a support. The block is raised a height \( h \) and then released. This block then strikes a block of mass \( M_2 \) on a frictionless surface. Find the velocity of the block \( M_2 \). Assume the collision is totally elastic.

2. Problem: Given the following set-up, why does only one of the balls swing out?

3. Problem: What force would need to be exerted by this linear catapult for the 30 Kg mass to have a muzzle velocity of 300 m/sec.

4. Problem: A 10 gram bullet strikes a 100 gram block of wood at rest with a velocity of 1000 m/sec. What is their combined velocity? Can you work the problem using the principle of conservation of momentum? Conservation of energy?

5. Problem: What is momentum and how does it relate to Newton's second law?

6. Problem: If you shoot a bullet into a ballistic pendulum, what was its initial velocity if you know the final height of the pendulum?

7. Problem: \( w = 100 \) lb

\[ V_0 = 100 \text{ ft/sec} \]

What is the maximum altitude reached?

8. Problem: A mass is dropped from a height \( H \). What is the velocity of the mass just before it hits the ground?

9. Problem: What are the velocities after an elastic collision?
10. Problem: Consider the following pendulum system:

a) If the bob is released from rest, what is the velocity as it passes the lowest point?

b) What assumptions are made in the answer?

c) What difference does it make if the system is in a vacuum?

d) Suppose a mass (m) were suspended at the lowest point, what would be the velocities of both masses after collision?

e) What if the collision were non-elastic?

11. Problem: Given a spring with the force-displacement relationship $F = e^x$, determine the energy required to move the block 3 units.


13. Problem: Given the velocities and masses of two objects, determine the kinetic energy and momentum of both masses before and after a collision in the case of elastic and inelastic bodies.

14. Problem: Find the velocity at point "A" for the cylinder shown.

15. Problem: Suppose you have the system shown below: Both friction and other dissipative forces apply.

a) Is the conservation of momentum theory applicable?

b) From what basic principle does the conservation of momentum theory come? Under what circumstances may it be applied?
II. STATIC

1. What is a moment arm?

2. Given the figure below, what conditions must be fulfilled if it is to remain in equilibrium? Where would the weight be placed to maintain equilibrium?

3. Find $F_1$ and $F_2$ given the figure below.

4. Given a block and tackle system and a known weight shown below, give the magnitude of the forces at the numbered points.
5. How much would the 1 Kg force raise the 50 Kg weight in the figure below?

6. Find the force at the contact points.
III. DYNAMICS

1. A sliding block slows from 16m/sec to 8m/sec in two seconds. If it weighs "B" Kg, what is the coefficient of sliding friction?

2. A car traveling at an initial velocity of $V_0$ applies its brakes to come to a stop. The coefficient of friction is $\mu$. What distance is required to stop?

3. A wheel rotating at an initial angular velocity of $\omega_0$ is accelerated to a final velocity of $\omega_f$ after 10 revolutions. What is the angular acceleration?

4. A person is standing on a building of height $h$. He throws a ball out with velocity $V_0$ at an angle to the horizon. How long will it take to reach the ground?

5. What is the velocity of block 2?

6. If the thread is pulled in the direction shown, which way will it roll?
7. Given the diagram below, what are the resultant force, velocity, and acceleration with respect to time?

\[ F_x \rightarrow 0 \]

8. What forces act on an object in orbit? Why does the object stay in the circular orbit?

9. What angle should a projectile be shot at in order to achieve maximum range?

10. How does acceleration relate to velocity?

11. A man is 8 meters from a bus. He is running 4 m/sec. The bus accelerates at 1 m/sec\(^2\). Does the man catch the bus?

12. State Newton's three laws.

13. What is the acceleration of the larger mass?

14. Given a particle in a circular orbit, give the horizontal component of its velocity. Give its position at any time \( t \) along the x-axis.

15. Given a coefficient of friction equal to \( \mu \), what force must be applied in order to move the block?

\[ \text{F} \]

16. Given a mass at a specified height with a horizontal velocity, how long will it take the mass to fall to the ground?

17. Derive \( F = ma \).
IV. FLUID DYNAMICS & BUOYANCY

1. Given a container with a cross-sectional area perpendicular to the y-axis of \(6 - \frac{y}{2}\), determine the pressure at its bottom. Water is flowing into the container at a rate of \((3 - t) \text{ ft}^3/\text{min}\) and leaving the container at a rate of \(t^2 \text{ ft}^3/\text{min}\). At time zero, the water level is two feet.

2. Describe how a Venturi meter works and show how it can be used to calculate fluid flow (Bernoulli's Equation and the Continuity Equation).

3. What happens to the water level with respect to the shore when the sailor throws the lead anchor overboard and it falls to the bottom of the lake?

4. How far will the water shoot out?

5. Given a cylindrical tank of diameter 30 cm and a cone with a base diameter of 12 cm and a height of 10 cm and a mass of 300 grams, how much would the water level in the tank rise when the cone is put into it? If you have forgotten the formula for the volume of a cone, how would you derive it using calculus?

6. Explain how you would estimate the flow rate in a pipe.
7. If the liquid is flowing in the direction indicated, where is the pressure the greatest?

8. Given a fluid flowing through a pipe in the direction indicated, what difference in parameters exists between points A and B?

9. What is meant by laminar flow? turbulent flow? If you had a piping system, is turbulent or laminar flow better? Why?

10. Which will raise the level of a tank of water higher when it is added - a one pound block of iron or a one pound block of wood?
V. WAVE PROPERTIES & OSCILLATIONS

1. What is the period of oscillation of a simple harmonic oscillator?

2. Derive the period of a simple pendulum.

3. Explain the difference between light and radio waves.

4. What is the relationship between frequency and wavelength?

5. What is the frequency of a 5Å wavelength wave?

6. Contrast light and sound waves. How do they propagate energy? Do they travel at different speeds in different media? Why?

7. Define Doppler Shift.

8. Put the following electro-magnetic radiation in order of increasing frequency: X-RAYS, Y RAYS, VISIBLE LIGHT, INFRARED


10. Draw a picture of a fish in water and show where you would throw a spear to hit the fish. Where does the fish appear? Why? How do n and C relate to refraction?

11. Draw a concave and a convex lens. What effect would each have on paraxial rays? Why?

12. 1) What does diffraction grating do, and what is it used for?

2) Are there circumstances under which light must be considered as a particle? Which ones?
VI. ELECTROMAGNETICS

1. What is capacitance and inductance?

2. Given a set of capacitor plates with voltage difference \( V \) a distance \( L \) apart in a vacuum, determine the velocity of a particle of mass \( m \) with charge \( q \) that starts at rest from one plate when it reaches the other plate.

3. How can the current in a wire be determined without touching the it?

4. Given the voltage \( V \), what is the current through resistor 3?

5. Given a RC circuit, determine \( I(t) \) and \( V(t) \) in both components, the energy stored in the capacitor, the energy dissipated by the resistor, and the total energy contributed from the battery.

6. Find \( I_R \) in the circuit below.

7. Graph \( I_1 \) and \( I_2 \) vs. time for the circuit below.
8. What is the average of voltage over one and one-half cycles?

9. Derive an experiment to measure the force on an electron in a magnetic field.

10. Discuss the pro and cons of three phase sources.

11. What is the current across a 300 watt lamp? \( V = 225 \, \text{v} \).

12. What is the difference between impedance and resistance?

13. Explain what happens to current when the switch is closed.

14. What are Maxwell's Equations used for?

15. Given a bar magnet, what does its field look like? Given a wire with electric current flowing in it, simulate the bar magnet's field. Can you, in a laboratory, simulate the earth's magnetic field?

16. What did Faraday do?

17. Are there some places on earth that might affect a ship's transmitters, receivers, etc. more than other places? Where?
18. Given three one ohm resistors, how many ways, using any number of the resistors (three maximum), can they be arranged to form different resistances?

19. Given three resistors in parallel. One resistor is 10 ohms, one is 20 ohms. What must be the value of the third resistor to form an equivalent circuit resistance of 6 ohms?

20. Given circuit 1 below, find I. Is I in figure 2 greater than or less than I in figure 1. Compare I for figure 3 with I for figure 1. What effect would an infinite or zero capacitance have in figure 3?

21. Explain how a simple motor works.

22. How would you go about proving that a group of series and/or parallel resistors are equivalent to a single resistor?
VII. ATOMIC

1. Explain the possible energy levels for x-rays (discrete or continuous).

2. What is the Schrodinger wave equation? What is it used for. What information about the Hydrogen atom can it be used to determine.


4. Given a substance (such as a plasma), which must be sustained at high temperature, how would you do it?

5. The half life of a compound is 40 years. After 60 years, what percentage of the compound has decayed away?
1. Why is glass more difficult to break when it has been tempered? Describe what occurs within the glass?

2. What is "Young's Modulus"? What is its approximate value for steel?

3. Describe the pros and cons of castings versus forgings.

4. Given an I beam. What types of forces are acting at the point of load on this beam? How do you find what these forces are?

5. Draw the shear and moment diagram for this beam.

6. Define the following and explain what quantities they compare. (also give units where applicable)

   Hooke's Law

   Poisson's Ratio

7. Define "cold working". What goes on within the material? What happens during "annealing"? Give physical properties.

8. Draw a stress-strain diagram for a substance such as steel. Where is the elastic limit? The yield point? What is stress and strain? How would work be defined on the stress-strain curve.

9. Draw Shear and Moment diagrams for this uniformly loaded beam.
1. How does a diode work?

2. What are the properties of a diode?

3. Describe in detail how a transistor works. Of what material is it constructed? How are impurity ions diffused into the material? What makes a transistor amplify?

4. Explain digital circuits.

5. Given the RLC circuit below, determine I(t) and graph it.

6. How do you find the Thevenin equivalent of a circuit?

7. Define Kirchhoff's laws.

8. Find the impedance for a series RLC circuit.

9. Find the equation for current in a series RL circuit and find the energy stored in the inductor.

10. Find the current in each branch of the circuit shown below.

11. Describe the difference between an AC and DC motor.

12. Describe the properties of synchronous and induction motors.
13. Given a separately-excited DC motor, how do you increase the speed? What is the reverse emf?

14. How does a transformer work? What sort of losses are involved?

15. Draw a capacitor and a resistor in parallel and input a sine wave. What type of current will flow?

16. Explain the operation of and diagram a full wave rectifier. Graph the input and output. How can you filter the output to look more like a DC source? What are the trade offs involved with this method? Using your method of filtering, do we lose maximum voltage or current?

17. Draw a simple oscillating circuit. What type of input would you use? What type of waveform would you get out?

18. Given the following AC delta circuit, what is $I_L$ if $R_1 = 0.5R_2 = 2R_3$? What is $I_L$ if all resistances are equal?

19. How does an electrical generator work?

20. How does impedance change with frequency? How does current change with frequency? Graph current versus frequency. At what frequency is the current at a maximum?

21. Explain what happens to current in a DC motor startup. What is the role of
22. Explain the physics of semi-conductors.

23. AC circuit. Draw V and I across L and C with respect to time.
1. Explain how a steam generator operates.

2. Given a full tank which holds 10,000 gallons, an empty barge with a capacity of 30,000 gallons, a pump and hose attached to the bottom of the tank car, and the barge 15 feet below the car, what should we do to empty the oil from the car to the barge? (Answer should include reference to air vents)

3. Explain the basic Rankine Steam cycle. Why is there a pump in the cycle?

4. Why is a counterflow heat exchanger more efficient than a crossflow heat exchanger?

5. Draw a gas turbine block diagram.

6. Both tanks are sealed. Water is in the upper tank. The lower tank is empty. Both valves are closed and the pump is off. How would you transfer water from the upper to the lower tank.

7. Knowing the work done by the steam flow, explain why this is not the power out of the nuclear reactor?
NUCLEAR ENGINEERING PROBLEMS

1. What is Buckling?
2. What does a pressurizer do? What is its function?
3. Draw a block diagram of a nuclear reactor.
4. Define fission.
5. What is criticality? What is the neutron multiplication factor? Write and explain the four factor formula.
6. What is binding energy? How does this relate to nuclear power?
7. What happens to the reactor as the temperature increases?
8. What is the difference between fission and fusion? How does a breeder reactor work?
9. Explain the difference between a pressurized water reactor and a boiling water reactor. Sketch a primary and secondary loop for a pressurized water reactor (PWR).
10. Describe how radiation is stopped by shielding. What types of material are best and why?
THERMODYNAMICS PROBLEMS

1. What are the first three laws of thermodynamics?

2. Describe in detail the Carnot and Rankine thermodynamic cycles.

3. If you know how much heat a bar is putting out, what is the temperature at the center of the bar?

4. What is enthalpy?

5. What is entropy? How is it used? Given two machines which perform the same task, but where machine "a" causes twice the entropy change that machine "b" does. Which would you rather use and why?

6. Suppose you have a container full of gas and you heat it up. What happens to the temperature and pressure? Why?

7. Given two boxes of identical volume and temperature filled with helium, is it possible to have different pressures?

8. You have 100 lbs. of water with c is constant. How much heat would be added to raise the temperature by 23 F?

9. What is temperature? What is heat content?

10. A rigid container is filled with a mixture of three gases, A, B, and C. The pressure gauge reads P_T. The container is evacuated and filled with an amount of gas equal to that in the mixture. The gauge reads P_A. The process is repeated for gases B and C. How is P_T related to P_A, P_B and P_C? What assumptions are made?

11. Draw a phase diagram for water. Show where solid, liquid, and gas lie. What is the "triple point"? Where is the region of five ices?

12. What thermodynamic property is associated with chemical processes? What is the definition of this quantity in terms of other thermodynamic variables?

13. Given the set-up in Figure 1, how much work will be done after part of the weight is removed (figure 2) knowing only the height the piston rises?
14. What variables affect the system temperature measurements?

15. a) If $\oint ds = 0$, what can you say about this process? 
b) If $\oint ds < 0$, what can you say about this process? 
c) If $\oint_1 ds < \oint_2 ds$, what can you conclude about these processes?

16. Suppose you have a container full of a gas and you heat it up. What happens to the temperature and pressure? Why?

17. What mathematical relationship is associated with the 2nd Law of thermodynamics?

18. Define entropy, enthalpy, and free energy. How is enthalpy measured? How is this used to calculate entropy?

19. Given a pipe with a heater and fluid velocity $V$, what happens to $T_2$ if $V$ doubles? What assumptions are made?

20. Determine the heat transfer and temperature profile across a wall of homogeneous material and thickness $L$. The fluid film heat transfer coefficients ($h$) are known.
21. What are the three modes of heat transfer? Discuss applicable equations.

22. Heat is added to a block of ice which is thermally isolated from its environment. Graph the temperature increase of a unit mass of ice per unit heat added as it changes phase to a vapor.

23. Explain the significance of the T-S diagram.

24. Given one pound of ice at $32^\circ F$, how much water at $200^\circ F$ must be added to bring the temperature to $50^\circ F$. Assume the specific heat of water to have a constant value of $1 \text{ BTU/(lb } {^\circ F})$.

25. What is a Mollier diagram?
CHEMISTRY

1. How many moles of carbon dioxide can be obtained from burning 3 moles of C3H3 in an abundance of oxygen?

2. State the mathematical expression for the pH factor of a solution. What is the pH of pure water? What happens to pH if the hydrogen ion concentration increases? How is the disassociation constant of water defined? What happens to the pH of water as temperature increases?

3. Give an example of the use of a differential equation in chemistry. Solve the equation.

4. Propose a method for detecting Cl⁻ in water.

5. My wife wanted to know how many molecules she was made of but she doesn't know any chemistry and you don't know what the body is made of. Assume she weighs 100 lbs and the body is made completely of water.

6. How do you determine if a reaction is exothermic or endothermic?

7. Draw the ammonia molecule and explain the forces acting on the atoms.

8. The half-life of a compound is 40 years. After 60 years, what percentage of the compound has decayed away?

9. Four liters of solution with pH 4 are added to six liters of solution with pH 6. What is the resultant pH?

10. Define "valence".

11. Given an inert container holding water with pH 7.0 open to the atmosphere, explain the chemical phenomena that take place.

12. Explain ionic bonding.

13. How do you determine the spontaneity of a chemical reaction?


15. What is an acid?

16. Draw an H₂ molecule. What types of bonds are involved? Why doesn't it fly apart?

17. How many atoms are there in 100 g of H₂SO₄?

18. Describe the distillation process of a substance. (example: water)

19. How many grams of iron will one liter of a 3 M solution of HCl dissolve?

20. What is a buffer solution?
SOLUTIONS TO MATHEMATICS PROBLEMS

I. GENERAL QUESTIONS

1. \(1 - 2y + y^2 + 2xy = 0\)

\[(1 - y)^2 = 2xy\]

\[\frac{(1 - y)^2}{2y} = x\]

Substituting the given solutions shows that (1, 1) is the correct answer.

2. b. hyperbola.

3. \(v_p = \frac{1}{3} (v_j)\)

\[t_p = t_j + 10\]

\[v_j t_j = 3000\]

\[v_p t_p = 3000\]

\[(\frac{1}{3}) (v_j) (t_j + 10) = 3000\]

\[(\frac{1}{3}) v_j t_j + (\frac{10}{3}) v_j = 3000\]

\[v_j = \frac{3}{10} (2000) = 600 \text{ mph}\]

4. \((x^2 - 2x + 1) + (y^2 - 4y + 4) = 17 + 5\)

\[(x - 1)^2 + (y - 2)^2 = 22\]

\[\therefore \text{ the center is (1, 2).}\]

5. \(\frac{y^2 - 1}{(y + 1)^2} \cdot \frac{1}{y - 1} = \frac{(y + 1)(y - 1)(y + 1)}{(y + 1)(y + 1)(y - 1)} = 1\)
6. \( a^4 + b^4 = \frac{a^4 + 2a^2b^2 + b^4 - 2a^2b^2}{a^2 + b^2} = \frac{(a^2 + b^2)^2 - 2a^2b^2}{a^2 + b^2} = a^2 + b^2 - 2a^2b^2 \)

\[ \frac{(3 + 21)(3 + 21)}{(3 - 21)(3 + 21)} = \frac{9 + 121 + 4i^2}{9 - 4i^2} = \frac{5 + 121}{13} \]

8. Fourier Series: \( f(x) = \sum_{n=1}^{\infty} \left( n g_n(x) = c_n g_1(x) + c_2 g_2(x) + \ldots \right) \)

Taylor Series: \( f(z) = \sum_{m=0}^{\infty} \frac{f^{(m)}(a)}{m!} (z - a)^m \)

Laplace Transforms: \( F(s) = \int_0^{\infty} e^{-st} f(t) \, dt = L(f) \)

\( f(t) = 1 \quad L(f) = 1/s \quad f(t) = e^{at} \quad L(f) = 1/(s-a) \)

\( f(t) = t \quad L(f) = 1/s^2 \)

\( f(t) = t^n \quad L(f) = n!/s^{n+1} \)

\( f(t) = \cos wt \quad L(f) = s^{2-}w^2 \)

\( f(t) = \sin wt \quad L(f) = w/s^2+w^2 \)

9. \( 9435_{10} = 22,333_{(8)} \)

10. \( 4361_{(8)} = \frac{8 + 384 + 1536 + 16384}{8} = 18312 = 2289_{10} \)
11. \[ P(A_j/B) = \frac{P(A_j) \cdot P(B/A_j)}{\sum_{j=1}^{k} P(A_j) \cdot P(B/A)} \]

12. Using Cramer's Rule:

\[ \begin{vmatrix} 5 & -4 & 2 \\ -3 & 4 & 0 \\ 1 & 0 & 4 \end{vmatrix} = 5(16) + 4(-12) + 2(-4) = 24 \]

\[ \begin{vmatrix} 0 & -4 & 2 \\ 6 & 4 & 0 \\ 6 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 0 & 2 \\ -3 & 6 & 0 \\ 1 & 6 & 4 \end{vmatrix} = 5(-4) - 4(0) + 2(6) = 1 \]

\[ \begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \frac{48}{24} = 2 \]
\[ \frac{72}{24} = 3 \]
\[ \frac{24}{24} = 1 \]

13. \( S(n) = (n)(n + 1) \)

14. Rational Root Theorem - possible real roots from factors of coefficient for \( X \).

Descartes' Rule of signs - # of positive or negative real roots based on sign changes \( f(x) \) and \( f(-x) \).

Derivatives for maxima, minima and curvature.

Function evaluation to estimate roots.

15. A logarithm is an inverse function of an exponential function, or more rigorously,

\[ \log_a = \{(x, y) \mid x = a^y, y \in \mathbb{R}, x \in (0, +\infty)\} \]
16. Let \( R \) = radius in feet

area in square feet = circumference in feet

\[
R^2 = 2R
\]
\[
R^2 - 2R = 0
\]
\[R(R - 2) = 0\]

\( R = 0 \) (not possible) \hspace{1cm} R - 2 = 0 \hspace{1cm} R = 2 \text{ feet}

17. A circle is defined as the set of ordered pairs \((x, y)\) such that the distance to another point is the same, i.e.

\[
r = (x - a)^2 + (y - b)^2
\]

where \((a, b)\) is the circle center and \( r \) is distance to points of the set from the center

\[
r^2 = (x - a)^2 + (y - b)^2
\]

18. 1) \( L = 2h \) \hspace{1cm} h = (1/2)L

2) \( w = L - 10 \)

3) \( 2Lw + 2Lh + 2hw = 10hw \)
\( 2Lw + 2Lh = 8hw \)

Substituting 1) and 2) into 3),

\[
2L(L - 10) + 2L(1/2) = 8(1/2(L))(L - 10)
\]
\[
3L^2 - 20L = 4L^2 - 40L
\]
\[
20L = L^2
\]

\( L = 0, 20 \)

dimensions are: \( L=20, \ w=10, \ h=10 \)

19. \( e = \lim_{x \to \infty} (1 + 1/x)^x \)
20. \( Ax^2 + Bx + C = 0 \)
\[
x^2 + Bx/A + C/A = 0
\]
\[
x^2 + Bx/A = -C/A
\]
\[
x^2 + Bx/A + B^2/4A^2 = B^2/4A^2 - C/A
\]
\[
(x + B/2A)^2 = (1/4A^2)(B^2 - 4AC)
\]
\[
x + B/2A = \frac{\sqrt{B^2 - 4AC}}{2A}
\]
\[
x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

21. The cross product for two vector functions in 3-space is a 3x3 determinant:
\[
\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
a & b & c \\
a' & b' & c'
\end{array}
\]

where \( a, a', b, b', c, c' \) are real-valued functions. The cross product is a vector orthogonal to both vectors.

The dot product of two vectors is a scalar resulting from the multiplication of the coefficients of each coordinate function \( \hat{i}, \hat{j}, \hat{k} \). The dot product of perpendicular vectors is 0.

22. A sphere encloses maximum volume with minimum surface area, a fact which can be verified using the appropriate formulas for various surfaces.
1. \( y = x^2 + 2x - 3 = (x + 3)(x - 1) \rightarrow (-3, 0) (1, 0) \) are roots
\[ y' = 2x + 2 = 0 \quad y' = 0 = 2x + 2 \rightarrow x = -1 \] is a relative min
\[ y'' = 2 \rightarrow \text{graph is concave up, since } y'' \text{ is always positive}. \]

2. A parabola of equation \( y = -x^2 + 4 \)

3. \( y = e^{-x^2} \)

4. \( y = e^{-2} \)
   \[ = 0.135 \]
5. \( y = e^{-x} \)

\[ y' = -e^{-x}, \text{ so there are no critical pts. (} y'' \text{ always negative) } \]

6. \( y = x^3 - x^2 \)

\[ y' = 3x^2 - 2x = 0 \]

\[ x(3x - 2) = 0 \quad \rightarrow \quad (0, 0) \text{ is a relative max, (} 2/3, -28/81 \text{) is a relative min by the first derivative test.} \]

\[ y'' = 6x - 2 \quad \rightarrow \quad (1/3, -2/27) \text{ is an inflection point.} \]
7. \( y = e^{-x^3} \)

\[
y' = -3x^2e^{-x^3} = 0 \quad \longrightarrow \quad (0, 1) \text{ is the only critical point; the first derivative test fails.}
\]

8. \( y = 12x^4 - 2x^3 + 8 \)

\[
y' = 48x^3 - 8x^2 = 0 \quad \longrightarrow \quad \text{critical points are at } x = 0 \text{ and } x = 1/6; \text{ the first derivative test shows that } x = 1/6 \text{ is a relative minimum.}
\]

\[
y'' = 144x^2 - 16x = 0 \quad \longrightarrow \quad \text{inflection points are at } x = 0 \text{ and } x = 1/9.
\]
11. \( y = x^x \)

NOTE: The graph is discontinuous at points where \( x \) is an even negative number.

12. \( f(x) = x^2 e^{-x^2} \)

\[
y' = 2xe^{-x^2} - 2x^3 e^{-x^2}
\]

\[= 2xe^{-x^2}(1 - x^2) = 0 \implies \text{relative maxima at } (1, 1/e) \text{ and } (-1, 1/e), \text{ a maximum at } (0, 1)\]

\[y'' = -4x^4 e^{-x^2} = 0 \text{ at } x = 0 \text{ only, so } (0, 1) \text{ is an inflection point.} \]
III. DIFFERENTIAL AND INTEGRAL CALCULUS

1. a) A differential is an incremental value of an independent variable (dx= ) or a function (dy = y'dx).
   b) A derivative is a limit of a different-quotient or \( f'(x) = \lim \left( \frac{y}{x} \right) \).
   c) The first derivative represents the slope of a function when evaluated at a point.
   d) The second derivative represents the curvature of a function when evaluated at a point.

2. Parts: \( \int u dv = uv - \int v du \)
   Chain Rule: \( \frac{d}{dx}(f(g(x))) = [f'(g(x))]g'(x) \)
   Division Rule: \( \frac{d}{dx}(f(x)/g(x)) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \)

3. An indefinite integral has no bounds, while a definite integral is bounded by continuous functions.

4. In two-space, an integral represents the area under the graph of the function and above an axis.

5. Integration is a summing process which determines a number
   \[ I = \int_a^b F(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} F[a + (i-1)\Delta x] \Delta x. \]

6. \( y = 2x + x^2e^{-x} \)
   \( y' = 2 + 2xe^{-x} - x^2e^{-x} \)

7. a) \( \frac{d}{dx}(x \sin^2 x = \sin^2 x + 2\sin x \cos x) \)
   b) \( \frac{d}{dx}(xe^{x^2} + x^3/\sin x - \sin^2 x + (x^2 + 1)^3) = e^{x^2} + 2xe^{x^2} + 3x^2/\sin x \)
   \( - x^3 \cos x \sin^2 x + 6x(x^2 + 1)^2 \)
   c) \( D_x(e^x) = e^x \)
   d) \( D_x \sin x = \cos x \)
e) \( D_x \cos x = - \sin x \)

f) \( D_x \ln x = \frac{1}{x} \)

g) \( D_x \cot x = - \csc^2 x \)

h) \( D_x \tan x = \sec^2 x \)

i) \( D_x \sech x = \sech x \tanh x \)

j) \( D_x \csch x = - \csch x \coth x \)

k) \( D_x 10^x = (10^x)(\ln 10) \)

l) \( D_y \left[ y^5 + (\cos y)e^y + \sin(y^2/3) \right] = 5y^4 - (\sin y)e^y + (\cos y)e^y + (2y/3)\cos(y^2/3) \)

m) \( D_x (x + x^3 + \sin x \cos x + \sin x) = 1 + 3x^2 + \cos^2 x - \sin^2 x + \cos x \)

n) \( D_y (y^2e^y) = 2ye^y + y^2e^y \)

8. \( V = (L - 2x)(w - 2x)(x) \)
\[ = Lwx - 2x^2w - 2x^2L + 4x^3 \]

\( V' = Lw - 4wx - 4Lx + 12x^2 \)

Set \( V' = 0 \) to find max and min:

\[ 12x^2 - (4w + 4L)x + Lw = 0 \]

Using the quadratic formula,

\[ x = \frac{(4w + 4L) \pm \sqrt{(4w + 4L)^2 - 48Lw}}{24} \]

To determine which point is the maximum, take the second derivative:

\( V'' = -4L - 4w + 24x \)  Evaluating for \( x \), \( V'' = + (4w + 4L)^2 - 48Lw \)

where \( V'' = 0 \), a maximum exists,

\[ . \ x = \frac{(4w + 4L) - \sqrt{(4w + 4L)^2 - 48Lw}}{24} \]
9. a) \( y = \sin x \)

\[ y' = \cos x \]

\[ y'' = -\sin x = 0 \] at point of inflection, \( \therefore x = 0 \).

b) \( y = x^4 - 2x^2 \)

\[ y' = 4x^3 - 4x \]

\[ y'' = 12x^2 - 4 = 0 \quad \implies x^2 = 1/3, \quad \therefore x = \pm\sqrt[3]{1/3} \]

So we have inflection points \( (\sqrt[3]{1/3}, -5/9), (\sqrt[3]{1/3}, -5/9) \).

10. L'Hopital's Rule is used when evaluating the limit of a function of the form

\[ Q(x) = \frac{F(x)}{G(x)} \]

produces an indeterminate form, e.g. \( 0/0 \). (\( F(x) \) and \( G(x) \) are continuous and differentiable.)

11. 1, using l'Hopital's Rule.

12. Evaluate \( y' = 0 \), solving for \( x \), and substitute into the original equation to find \( y \). This point is a maximum or a minimum. Evaluate \( y'' \); if positive, the function is concave up and the point is a minimum; if negative, the function is concave down and the point is a maximum.

13. a) rectangular area:

\[ L + 2w = 80 \]

Area = \( Lw \)

\[ L = 80 - 2w \]

\[ A = (80 - 2w)w \]

\[ = 80w - 2w^2 \]

\[ dA = 80 - 4w = 0 \at \text{max area} \implies w = 20, \quad L = 40 \]

\[ \text{Area} = 800 \text{ ft}^2 \]
b) half-circle

\[ \pi r = 80 \]
\[ A = \frac{\pi r^2}{2} \]
so \( r = \frac{80}{\pi} \)

\[ A = \frac{\pi}{2} \left( \frac{80}{\pi} \right)^2 \]
\[ = \frac{1}{2\pi} (6400) \]
\[ = 3200/\pi \]
\[ A = 1000 \text{ ft}^2 \]

" a half-circle will provide the greatest area.

14. \( D_x x^2 = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \)

\[ = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \]
\[ = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x} \]
\[ = \lim_{\Delta x \to 0} (2x + \Delta x) \]
\[ = 2x \]

15. \( y = 1 + e^{-x} \) first derivative is always negative --> no maxima or minima

\[ y' = -e^{-x} \]
second derivative is always positive --> no inflection points

\[ y'' = e^{-x} \]

16. Integration is the summation of infinitely many rectangles of dimensions \( f(x) \) and \( dx \) under a curve \( y = f(x) \).

17. a) \( \int \sec^2 x \, dx = \tan x + C \)

b) \( \int (2x + 1) \, dx = x^2 + x + C \)

c) \( \int (y + 3)(y + 1) \, dy = \int (y^2 + 4y + 3) \, dy \]
\[ = \frac{y^3}{3} + 2y^2 + 3y + C \]

d) \( \int x \sin x \, dx = \sin x - x \cos x + C \)

e) \( \int \tan^2 (x) \cos(x) \, dx = \tan^2 (x) \cos(x) + C \)
f) \[ \int x^{\sqrt{a^2 - x^2}} \, dx = -\frac{1}{2}\int -2x \, \sqrt{a^2 - x^2} \, dx \]
\[ = \frac{2}{3}(a^2 - x^2)^{3/2}(-1/2) + C \]
\[ = -\frac{1}{3}(a^2 - x^2)^{3/2} + C \]

g) \[ \int (x^2 + xe^{-x}) \, dx = x^3/3 - xe^{-x} - e^{-x} + C \]
h) \[ \int e^{-x} \, dx = -e^{-x} + C \]
i) \[ \int_{0}^{1} (x + 2) \, d(x + 1) = \int_{0}^{1} (x + 1 + 1) \, d(x + 1) \]
\[ = \int_{0}^{1} (x + 1) \, d(x + 1) + d(x + 1) \]
\[ = \int_{0}^{1} \frac{(x + 1)^2}{2} + x + 1 \]
\[ = (2 + 2) - (1/2 + 1) \]
\[ = \frac{5}{2} \]

j) \[ \int xe^x \, dx = xe^x - \int e^x \, dx \]
where \[ x = u \]
\[ dv = e^x \, dx \]
\[ \int u \, dv = uv - \int v \, du \] (integration by parts)
\[ = xe^x - e^x \]
\[ dx = du \]
\[ v = e^x \]
\[ = e^x(x - 1) \]

k) \[ \int_{0}^{\pi/2} \int_{0}^{\pi/2} r \sin \theta \, d\phi \, d\theta \, dr = \int_{0}^{\pi/2} \int_{0}^{\pi/2} [\phi r \sin \theta]_{0}^{\pi/2} \]
\[ = \frac{\pi}{2} \int_{0}^{\pi/2} r \sin \theta \, d\theta \]
\[ = \frac{\pi}{2} \int_{0}^{\pi/2} r [-\cos \theta]_{0}^{\pi/2} \, dr \]
\[ = \frac{\pi}{2} \int_{0}^{\pi/2} r \, dr \]
\[ = \frac{\pi}{2} [r^2/2]_{0}^{\pi/2} \]
\[ = \frac{\pi R^2}{4} \]

1) \[ \int \sec(u) \tan(u) \, du = \sec(u) + C \]
m) \[ \int \csc^2(u) \, du = -\cot(u) + C \]
n) \[ \int \sec^2(x) \, dx = \tan(x) + C \]
o) \[ \int \cos(x) \, dx = \sin(x) + C \]
20. \[ A = \int_{-2}^{2} (4 - x^2) \, dx \]
\[ = 4x - \frac{x^3}{3}\bigg|_{-2}^{2} \]
\[ = 4(2 - (-2)) - \left( \frac{8}{3} - \left(-\frac{8}{3}\right) \right) \]
\[ = 16 - \frac{16}{3} \]
\[ = \frac{32}{3} \]

21. \[ V = \int_{0}^{H} \int_{0}^{2} \int_{0}^{R(H - z)/H} r \, dr \, dz \]
\[ = \int_{0}^{H} \int_{0}^{2} \left( \frac{r^2}{2} \right) \frac{R(H - z)}{H} \, dz \]
\[ = \int_{0}^{H} \frac{R^2}{2} \left( \frac{(H - z)^2}{H^2} \right) \, dz \]
\[ = \left( \frac{R^2}{2H^2} \right) \left( H^3 - 2H^2z + z^3 \right) \bigg|_{0}^{H} \]
\[ = \frac{R^2}{H} \left( H^3 / 3 \right) \]
\[ V = \frac{\pi R^2 H}{3} \]

From similar triangles:
\[ \frac{H - z}{H} = \frac{r}{R} \]
\[ \therefore r = R \frac{H - z}{H} \]

22. Find the volume in first quadrant using a triple integral and multiply by 4.

\[ \int_{0}^{H} \int_{0}^{W/2} \int_{0}^{x} dx \, dy \, dz \]

Determine limits of integration.
\[ z = mx + b = -\left[ \frac{H}{(W/2)} \right] x + H \]
\[ \therefore x = \left( z - H \right) \left( -\frac{W}{2H} \right) \]
\[ = \left( \frac{W}{2} \right) \left( 1 - z/H \right) \]

Also \( y = \left( \frac{W}{2} \right) \left( 1 - z/H \right) \)

\[ V = 4 \int_{0}^{H} \int_{0}^{W/2} \int_{0}^{x} dx \, dy \, dz \]
p) \( \int \sin(x) \, dx = -\cos(x) + C \)
q) \( \int \sec(x)\tan(x) \, dx = \sec(x) + C \)
r) \( \int (2x + 1) \, dx = x^2 + x + C \)
s) \( \int dx/x = \ln x + C \)
t) \( \int x^2 \sin(x) \, dx = 2x \sin(x) + 2\cos(x) - x^2 \cos(x) \) (integration by parts)
u) \( \int e^x \sin(x) \, dx = e^x \cos(x) + e^x \sin(x) + C - \int e^x \sin(x) \, dx \)
\( = e^x [\sin(x) - \cos(x)]/2 + C \) \quad \text{where} \quad C = 2C_1
v) \( \int \cos^2(x) \, dx = 1/2 \int (1 + \cos(2x)) \, dx \)
\( = x/2 + \sin(2x)/4 + C \)

18. As \( d\theta \) approaches \( \theta \), the area of the shaded sector approaches that of a triangle with base \( Rd\theta \) and height \( R \). Area of triangle is \( 1/2 \) (base)(height) or \( 1/2 \) (\( R\theta \) \( R \)). Summing up the areas of the triangles, yields the integral:

\[
\int_0^{2\pi} \frac{1}{2} (R^2 \, d\theta) = \pi R^2
\]

19. \( y = -hx/b + h \)

\[
A = \int_0^b y \, dx = \int_0^b (-hx/b + h) \, dx
\]
\( = -hx^2/2b \bigg|_0^b + hx \bigg|_0^b \)
\( = -hb^2/2b + hb \)
\( = -hb/2 + hb \)

\( A = hb/2 \)
\[ \begin{align*}
&= 4 \int_0^H \int_0^y x \, dy \, dz \\
&= 4 \int_0^H \int_0^{(W/2)(1 - z/H)} y \, dy \, dz \\
&= 4 \int_0^H (W/2)^2 (1 - z/H)^2 \, dz \\
&= W^2 \int_0^H (1 - 2z/H + z^2/H^2) \, dz \\
&= W^2 \left( z - z^2/H + z^3/(3H^2) \right)_0^H \\
&= W^2 (H - H^2/H + H^3/(3H^2)) \\
&= W^2 (H - H + H/3) \\
&= W^2 H/3
\end{align*} \]

23. As \( dr \) and \( d\theta \) approach \( 0 \), the shaded area approaches that of a rectangle with area \((r \, d\theta \, dr)\).

The area of the shaded section is \((R \, d\theta \, dz)\). Integrate to sum the area from \( \theta \) to \( 2\pi \) and \( z \) from \( z \) to \( H \).

\[ A = \text{area of circles} + \text{area of sides} \]

\[ = 2 \int_0^{2\pi} \int_0^R r \, dr \, d\theta + \int_0^H \int_0^{2\pi} R \, dz \, d\theta \]
\[ \int_{0}^{2\pi} R^2/2 \, d\theta + \int_{0}^{H} 2\pi R \, dz \]
\[ = 2\pi R^2 + 2\pi R H \]

24. Upperbound
\[ 4 - x^2 = x^2 - 1 \]
\[ 2x^2 = 5 \]
\[ x^2 = 5/2 \]
\[ x = \pm\sqrt{5/2} \]

Area (shaded) = \[ \int_{0}^{1} (4 - x^2)dx + \sqrt{5/2} \left( 4 - x^2 \right) - (x^2 - 1) \, dx \]
\[ = \int_{0}^{1} (4 - x^2)dx + \sqrt{5/2} \left( 5 - 2x^2 \right) \, dx \]
\[ = \left[ 4x - x^3/3 \right]_{0}^{1} + \left[ 5x - 2x^3/3 \right]_{0}^{5/2} \]
\[ = 10/3 \sqrt{5/2} - 2/3 \]

25. let \( m = \text{mass per unit volume} \)
\[ \bar{y}m_t = [(a)(1)(1)m(-a/2)] + [(a)(1-a)(1)m](1-a)/2 \]
\[ \bar{y}[(a)(1)(1)m + (a)(1-a)(1)m] = -a^2m/2 + (a-2a^2 + a^3)m/2 \]
\[ \bar{y}(a + a -a^2) = (a - 3a^2 + a^3)/2 \]
\[ \bar{y} = (a - 3a^2 + a^3)/(4a - 2a^2) \]
\[ \bar{y} = (a^2 - 3a + 1)/(4 - 2a) \]
26. Integrate \( A = \int_{0}^{2} y \, dx \)

\[
= \int_{2}^{0} (x^2 + x - 6) \, dx
\]

\[
= -x^3/3 - x^2/2 + 6x \bigg|_{2}^{0}
\]

\[
= 22/3
\]

27. \( y_1 = x^2 \\
    y_2 = x^3 \)

\[
A = \int_{0}^{1} (y_1 - y_2) \, dx
\]

\[
A = \int_{0}^{1} (x^2 - x^3) \, dx
\]

\[
A = x^3/3 - x^4/4 \bigg|_{0}^{1}
\]

\[
A = 1/3 - 1/4
\]

\[
A = 1/12
\]

28. \( A = \int_{0}^{1} x \ dy \ dx \)

\[
A = \int_{0}^{1} y \int_{0}^{x^2} \, dx
\]

\[
A = \int_{0}^{1} (x - x^2) \, dx
\]
26. Integrate \( A = \int_{0}^{1} y \, dx \) \\
\[ = \int_{0}^{1} (x^2 + x - 6) \, dx \]
\[ = -x^3/3 - x^2/2 + 6x \bigg|_{0}^{1} \]
\[ = 22/3 \]

27. \( y_1 = x^2 \)
\( y_2 = x^3 \)
\[ A = \int_{0}^{1} (y_1 - y_2) \, dx \]
\[ = \int_{0}^{1} (x^2 - x^3) \, dx \]
\[ A = x^3/3 - x^4/4 \bigg|_{0}^{1} \]
\[ A = 1/3 - 1/4 \]
\[ A = 1/12 \]

28. \( A = \int_{0}^{1} x \frac{dy}{dx} \, dx \)
\[ = \int_{0}^{1} x^3 \, dx \]
\[ = \frac{1}{4} \left[x^4\right]_{0}^{1} \]
\[ A = \int_{0}^{1} (x - x^2) \, dx \]
A = \frac{x^2}{2} - \frac{x^3}{3}]_0^1

A = \frac{1}{2} - \frac{1}{3}

A = \frac{1}{6}

dV = \pi R^2 H

dV = \pi y^2 \, dx

V = \int_1^\infty \pi y^2 \, dx

= \int_1^\infty \pi/x^2 \, dx

= -\pi/x]_1^\infty

= (0 - (-\pi))

Vol = \pi

30. Using polar coordinates,

A = \int_0^{2\pi} \int_0^1 r \, dr \, d\theta

A = \int_0^{2\pi} r^2/2]_1^2 \, d\theta

A = 3/2 \int_0^{2\pi} \, d\theta

A = 3\pi

85
1. Assume \( x = C_1 e^{-ct} \)

then: \( x' = CC_1 e^{-Ct} \)
\( x'' = C^2 C_1 e^{-Ct} \)

Substituting into original equation for \( x, x', x'' \):
\( C^2 C_1 e^{-Ct} - 5CC_1 e^{-Ct} + C_1 e^{-Ct} = 0 \)

dividing by \( C_1 e^{-Ct} \):
\( C^2 - 5C + 6C = 0 \)

\( (C - 3)(C - 2) = 0 \)

\[ \therefore \ C = 2, 3 \]

Thereby the general solution is:
\( x = C_1 e^{-Ct} + C_2 e^{-Ct} \)

To obtain the particular solution assume:
\( x = C_1 e^{-t} \)

then: \( x' = -C_1 e^{-t} \)
\( x'' = C_1 e^{-t} \)

Substituting into the original equation:
\( C_1 e^{-t} - 5C_1 e^{-t} + 6C_1 e^{-t} = e^{-t} \)

Dividing by \( e^{-t} \):
\( C_1 - 5C_1 + 6C_1 = 1 \)

\( 2C_1 = 1 \)

\( C_1 = \frac{1}{2} \)

Therefore, the particular solution is:
\( x = C_1 e^{-2t} + C_2 e^{-3t} + e^{-t/2} \)
2. \( F(t) = t \)

\[
L_f(s) = \int_0^\infty f(t) e^{-st} dt \\
= \int_0^\infty te^{-st} dt \\
= [te^{-st}]_0^\infty + (1/s) \int_0^\infty e^{-st} dt \\
= 0 + (-1/s^2) [e^{-st}]_0^\infty \\
= -1/s^2(0 - 1) \\
L(t) = 1/s^2
\]

3. GENERAL SOLUTION for \( y'' + 4y' + 3y = 0 \)

Assume \( y = e^{-cx} \)

\[
y' = -ce^{-cx} \\
y'' = c^2 e^{-cx}
\]

Substituting:

\[
c^2 e^{-cx} - 4ce^{-cx} + 3e^{-cx} = 0 \\
(c - 3)(c - 1) = 0 \\
\]

\( c = 1 \) or \( 3 \)

General solution: \( y_c = e^{-x} + e^{-3x} \)

PARTICULAR SOLUTION for \( y'' + 4y' + 3y = 0 \)

Assume \( y = Asin(x) + Bcos(x) \)

\[
y' = Acos(x) - Bsin(x) \\
y'' = -Asin(x) - Bcos(x)
\]

Substituting:

\[-Asin(x) - Bcos(x) + 4Acos(x) - 4Bsin(x) + 3Asin(x) + 3Bcos(x) = \sin(x)\]
\((-A - 4B + 3A)\sin(x) + (-B + 4A + 3B)\cos(x) = \sin(x)\)

\[2A - 4B = 1\quad 4A + 2B = 0\]
\[4A = -2B\]
\[2A - 4(-2A) = 1 \quad \text{----} \quad B = -2A\]
\[10A = 1\quad B = -1/5\]

Particular Solution: \(y = e^{-x} + e^{-3x} + (1/10)\sin(x) - (1/5)\cos(x)\)

4. \(dx/dt = x/K\)
\[K \frac{dx}{dt} = dt\]
\[dt = K \frac{dx}{x}\]
\[t = K \ln x\]
\[t/K = \ln x\]
\[e^{\frac{t}{K}} = x\]
\[x = e^{\frac{t}{K}}\]

5. \(y' + Ky = 10\)

General Solution
\[y' + Ky = 0\]
\[dy/dx = -Ky\]
\[\int dy/y = \int -K \, dx\]
\[\ln y = -Kx\]
\[y = e^{-Kx}\]

Specific Solution
\[y' + Ky = 10\]
Assume \(y = 10/K\)
\[y' = 0\]
\[0 + K(10/K) = 10\]
\[10 = 10\]

\[\therefore \text{Solution: } y = e^{-Kx} + 10/K\]

6. Assume a solution with undetermined coefficients \((C_i)\), differentiate the solution for as many derivatives exist, and substitute into the original equation to determine values for coefficients.

The solution would be of the form \(A = B\sin(x) + C\cos(x)\) or imaginary exponentials.
7. $y = 3y' = 0$
   $y - 3 \frac{dy}{dx} = 0$
   $3 \frac{dy}{dx} = y$
   $3 \frac{dy}{y} = dx$
   $3 \ln y = x + C$
   $\ln y^3 = x + C$
   $y^3 = e^{x + C}$
   $y = e^{(x + C)/3}$
   $y = e^{x/3} e^{C/3}$
   $y = C_1 e^{x/3}$
   $y = 3C_1 e^{x/3}$  \hspace{1cm} \text{let } C_1 = e^{C/3}$
   $y = 3C_1 e^{x/3}$

V. PROBABILITY AND STATISTICS

1. Solution: \# of seating arrangements for three men = $A = 3P_3 = 3! = 6$
   \# of consecutive three seats is a function of 1 seat over 5 seats or:
   \[ B = 5P_1 = \frac{5!}{(5-1)!} = \frac{5!}{4!} = 5 \]
   Total = $(A)(B) = (6)(5) = 30$

2. The mean is the arithmetic average of the data. The median is the value such that an equal number of data is greater than and less than it. The mode is the data that occurred with the greatest frequency. All three are measures of central tendency.

3. A Gaussian distribution is also a normal distribution or the "bell-shaped curve". It density function is
   \[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2} \]
   where $\mu$ is the mean and $\sigma^2$ is the variance.
4. Solution: \[ \frac{5!}{2! (5-2)!} \cdot \frac{(1)(2)(3)(4)(5)}{(1)(2)(1)(2)(3)} = 10 \]

5. a. After the first pick, there are 49 red balls and 50 black balls remaining. Therefore, 99 total balls with 49 of those red. Thus, 49/99.

   b. In a like manner, 50/99.

NOTE: For a check, all possibilities must add up to 1. Also, the events are independent of one another.
1. \( E_p + E_{k1} = E_p + E_{k2} \)

Determine the maximum velocity of \( m_1 \) using conservation of energy.

Energy of \( m_1 = E_p = m_1 gh = E_k = (1/2)m_1 v_1^2 \)

1. \( m_1 gh = (1/2)m_1 v_1^2 \)

\[ v_1 = \sqrt{2gh} \]

Conservation of momentum and kinetic energy at collision

2. \( m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \)

3. \( (m_1 v_{1i}^2 + m_2 v_{2i}^2)/2 = (m_1 v_{1f}^2 + m_2 v_{2f}^2)/2 \)

Solve three equations and three unknowns for \( v_{1f} \) and \( v_{2f} \)

Place 1. into 2. and solve for \( v_{1f} \)

\[ v_{1f} = \sqrt{2gh - \frac{m_2}{m_1} v_{2f}^2} \]

Place 1. into 3. and solve for \( v_{1f} \)

\[ v_{1f} = \sqrt{2gh - \frac{m_2}{m_1} v_{2f}^2} \]

Solve for \( v_{2f} \) by setting the above equations equal to each other

\[ (2gh - \frac{m_2}{m_1} v_{2f}^2) = (2gh - \frac{m_2}{m_1} v_{2f}^2) \]

\[ 2gh - 2\sqrt{2gh m_2 v_{2f}/m_1} + m_2 v_{2f}^2/m_1 = 2gh - m_2 v_{2f}^2/m_1 \]

\[ \sqrt{2gh m_2/m_1} = v_{2f} \quad (m_2/m_1 + m_2^2/m_1^2) \]

\[ v_{2f} = \frac{2\sqrt{2gh m_2/m_1}}{m_1^2 + m_2/m_1} = \frac{2\sqrt{2gh}}{(1+m_2/m_1)} \]

\[ v_{1f} = \frac{m_1 \sqrt{2gh} - m_2 2\sqrt{2gh}/(1+m_2/m_1)}{m_1} \]

\[ v_{1f} = \frac{2gh - m_2}{m_1} \left( \frac{m_2}{m_1} \right) \left( \frac{m_1}{m_2} \right)^2 \]

\[ = \frac{2gh (1 - 2/(m_1/m_2+1))} {m_1} \]

2. Due to the laws of conservation of energy and momentum, Ball A transfers its momentum and energy to the inner spheres, each one transferring them to the next sphere. Because no sphere is located next to it, sphere B retains the momentum and kinetic energy and moves.

3. Conservation of Energy
\[
\frac{1}{2}mv^2 = \int F \, dx
\]
Assuming a constant force \( F \)
\[
\frac{1}{2}mv^2 = F \Delta x
\]

\[
F = \frac{mv^2}{\Delta x} = \frac{(30 \text{ kg})(300 \text{ m/s})^2}{2(300 \text{ m})} = 4500 \text{ Newtons}
\]

4. 1. Conservation of Momentum (assume no friction)

\[
m_1v_1 = (m_1 + m_2)v
\]

\[
v = \frac{(m_1v_1)}{(m_1 + m_2)}
\]

\[
v = \frac{(10)(1000 \text{ m/sec})}{110} = 91 \text{ m/sec}
\]

2. Conservation of momentum is used if negligible air friction occurs.

3. Conservation of energy may not be used since the absorption of the bullet into the block of wood is energy loss from the system.

5. Momentum \((p)\) is the product of the mass of a body and its velocity. Newton's second law may be stated in terms of momentum:

\[
F = ma \quad p = mv
\]

\[
F = m \frac{dv}{dt}
\]

\[
= d(mv)/dt
\]

\[
F = dp/dt
\]

6. Momentum is conserved in the collision of the bullet with the pendulum.

\[
mv_B = (M+m) \, v_0
\]

\[
v_0 = (m)V_B/(M+m)
\]

Consider Energy change of pendulum:

\[
E_k_1 + E_{p1} = E_{k2} + E_{p2}
\]

\[
(M+m)v_0^2/2 + 0 = 0 + (M+m)gh
\]

\[
(M+m)m^2v_B^2/2(M+m)^2 = (M+m)gh
\]

\[
v_B^2 = (M+m)^2(2gh)/m^2
\]

\[
v_B = (M+m)\sqrt{2gh/m}
\]
7. Assume negligible air resistance

\[ \ddot{a} = \frac{dv}{dt} \cdot \frac{dv}{dx} \cdot \frac{dx}{dt} \]

\[ \ddot{a} = v \cdot \frac{dv}{dx} \]

\[ \int \ddot{a} \, dx = \int v \, dv \]

\[ a \Delta x = v_f^2/2 - v_0^2/2 \]

\[ v_f^2 = v_0^2 + 2g \cdot x \]

\[ 0 = v_0^2 - 2gy \]

\[ y = \frac{v_0^2}{2g} + (100)^2/2(32) = 156 \text{ ft} \]

or Conservation of Energy

\[ E_k = E_p \]

\[ mv_0^2/2 = mgy \]

\[ y = \frac{v_0^2}{2g} = (100)^2/2(32) = 156 \text{ ft} \]

8. \[ v_f^2 = v_0^2 + 2ax \]

\[ v_f^2 = (0)^2 + 2(32) \cdot H \]

\[ v_f^2 = 64 \cdot H \]

\[ v_f = 8 \sqrt{H} \]

9. Conservation of Momentum:

\[ m_1 - \frac{1}{2} m_2 = m_2 v_2 - m_1 v_1 \]

Conservation of Energy

\[ \frac{m_1}{2} + \frac{m_2}{2} = m_2 \frac{v_2^2}{2} + m_1 \frac{v_1^2}{2} \]

\[ v_2 = (m_1 - \frac{m_2}{2} + m_1 v_1)/m_2 \]

Solve above 2 equations for \( v_1 \) & \( v_2 \) in terms of \( m_1 \) & \( m_2 \)

Divide Through by \( m_1 \) (2\( m_1 = m_2 = 2 \text{ kg} \))

\[ 3/4 \ v_1^2 = 3/4 \]

\[ v_1 = 1 \]

\[ v_2 = 1/2 \]
10. a) \( \frac{mv^2}{2} = mg\Delta H \quad v = \sqrt{2gH} \)

b) Assumptions - no external, non-conservative forces (friction) act on the system. The string used is weightless and in-extensible.

c) The only difference is the loss of an air friction force which is a negligible effect.

d) With masses of the same quantity, all the energy of the first ball would be transferred to the second ball, assuming the collision to be totally elastic.
\[ v_2 = \frac{(m) g \Delta H}{v_1} = 0 \]
e) For the totally inelastic collision with equal masses
\[
m_1v_1 = (m_1 + m_2)v_2
\]
\[
v_2 = \frac{m_1v_1}{m_1 + m_2} = \frac{m\sqrt{2gH}}{m_1 + m_2}
\]

11. \[ W = \int F \, dx \quad \text{where } F = e^x \]
\[
= \int e^x \, dx = e^x + c
\]
\[
= e^x - e^0
\]

12. The basic definition of work is the product of the force applied to an object, the magnitude of the displacement of the object and the cosine of the angle between them (\( W = \int F \cdot dr \)). Energy is the ability to do work. Power is a measure of how fast work is being done or energy is being produced.

13. Assume one dimensional case.
Conservation of Momentum: \( (m_1v_1 + m_2v_2)_i = (m_1v_1 + m_2v_2)_f \)
Conservation of Energy (elastic case only):
\( (m_1v_1^2 + m_2v_2^2)_i = (m_1v_1^2 + m_2v_2^2)_f \)
Coefficient of restitution \( e = \frac{(v_2 - v_1)_f}{(v_1 - v_2)_i} \)
e = 1 in the case of an elastic collision

Determine \( v_1 \) after collision:
\[
m_1v_{1f} = m_1v_{1i} + m_2v_{2i} - m_2v_{2f}
\]
\[
v_{2f} = e(v_{1i} - v_{2i}) + v_{1f}
\]
\[
v_{1f} = (v_{1i} + v_{2i}m_2/m_1 - m_2/m_1(e(v_{1i} - v_{2i}) + v_{2f}))
\]
\[
v_{1f} = \frac{(m_1v_{1i} + m_2v_{2i})/m_1 - (v_{1i} - v_{2i})m_2/m_1}{(m_2 + m_1)/m_1}
\]
\[ v_{1f} = \left( m_1 v_{1i} + m_2 v_{2i} - m_2 e (v_{1i} - v_{2i}) \right)/(m_1 + m_2) \]

Then \( v_{2f} = e(v_{1i} - v_{2i}) + v_{1f} \)

The momentum and kinetic energies of both masses, before and after the collision will be \( mv \) and \( 1/2 mv^2 \) respectively.

14. Energy Balance

\[ E_{k1} + E_{p1} = E_{k2} + E_{p2} \]
\[ mv^2/2 + Iw^2/2 + mgh = m_{v1f}^2/2 + Iw_{2f}^2/2 \]

I for the cylinder is \( mr^2/2 \)
\( \nu = wr \) assuming no slippage

\[ Iw^2/2 = (mr^2/2)v^2/r^2) = mv^2/4 \text{ and} \]
\[ 3mv^2/4 + mgh = 3mv_{2f}^2/4 \]

\[ v_{2f} = \sqrt{\frac{v_1^2 + 4gh/3}{4m^2/s^2 + 4(9.81)(2)/3 m^2/s^2}} \]
\[ = \sqrt{30.2} = 5.5 \text{ m/s} \]

15. a) Since sliding friction and air resistance are external forces on the system, the momentum of the system is not conserved.

b) Given a system of objects, the forces acting on any of the objects can be classified as external and internal. Internal forces occur when one of the objects within the system strikes another. Only when the vector of all external forces acting on the system is zero is momentum conserved.
I. STATICS SOLUTIONS

1. A moment arm is the perpendicular distance between an axis point and the line of action of a force.

\[ \sum F_y = 0 \quad \therefore 10 + 15 - F_1 - F_2 = 0 \]
\[ \sum F_x = 0 \quad \therefore \text{none} \]
\[ \sum F_z = 0 \quad \therefore \text{To determine the maximum value of } \alpha, \text{ set } F_1 \text{ equal to zero.} \]
\[ (a - 6')15 \text{ lb} - (6' - 5')10 \text{ lb} = 0 \]
\[ (a - 6')15 \text{ lb} = 10 \text{ lb} \]
\[ a = 6' = 10/15 \]
\[ a = 6 \frac{2}{3} \text{ ft} \]
\[ \therefore 0 \leq a \leq 6 \frac{2}{3} \]

3. \[ \sum T_{F_1} = 0 \]
   \[ \therefore \text{d}F_2 - (d/2)W_1 - (2d/3)W_2 = 0 \]
   \[ \therefore F_2 = (1/2)W_1 + (2/3)W_2 \]
\[ \sum F_y = m a = 0 \]
   \[ \therefore |F_1| + |F_2| - |W_1| - |W_2| = 0 \]
   \[ F_1 + (1/2)W_1 + (2/3)W_2 - W_1 - W_2 = 0 \]
   \[ F_1 = (1/2)W_1 + (1/3)W_2 \]

4. Assume system acceleration is 0. \[ T_{\text{cp}} = 0 \] (cp=center of pulley)

Pulley 3
   \[ r_1 \]
   \[ r_2 \]
   \[ r_3 \]
   \[ F_3 = 2T \]
   \[ = 200 \text{ lbs} \]
   \[ \therefore T_1 = T_2 = 100 \text{ lbs} \]

Pulley 2
   \[ r_1 \]
   \[ r_3 \]
   \[ F_2 = 2T \]
   \[ = 200 \text{ lbs} \]
   \[ \therefore T_1 = T_2 = 100 \text{ lbs} \]

Pulley 1
   \[ r_4 \]
   \[ r_3 \]
   \[ F_3 = 2T \]
   \[ = 400 \text{ lbs} \]
   \[ \therefore T_1 = T_2 = 100 \text{ lbs} \]
5. The force on the line will displace it a distance proportional to the magnitude of the force. The distance the weight is raised is equal to the length of the deformed line minus its original length.

\[ \Delta x = 2x - 2L \]

\[ = 2(\sqrt{L^2 + (L/50)^2}) - 2L \]

\[ = 2\sqrt{L} - 2L \]

\[ = 0.0004L \]

6. Summation of the moments about A is equal to zero.

\[
\text{\because } W(2/3 d) - F_B(d) = 0
\]

\[ F_B = 2/3 W \]

\[ F_A = 1/3 W \]
II. DYNAMICS SOLUTIONS

1. \[
F_s = \mu N
\]
\[
F_s = ma
\]
\[
A mg = m \Delta v / \Delta t
\]
\[
A = \Delta v / (\Delta t \ g) = [(16 - 8) m/s] / [(2 \ sec)(9.8 \ m/s^2)] = 0.408
\]

2. \[
F = ma = mg \mu_e
\]
\[
v_p^2 = v_0^2 + 2as
\]
\[
0 = v_0^2 + 2(-g \mu_e) s
\]
\[
2g \mu_e s = v_0^2
\]
\[
s = v_0^2 / 2g \mu_e
\]

3. \[
\alpha = \Delta \theta = \Delta (W_\theta / 2) - (W_1 / 2)
\]
\[
W_\theta = W_0 + 2\alpha \theta
\]
\[
\alpha \theta = W_\theta - W_0
\]
\[
\alpha = (W_\theta - W_0) / 40 \pi
\]

4. Position in the y direction is: \[
y = v_0 t \sin \theta - gt^2 / 2 + h
\]

Set \[y = 0\]
\[
(gt^2) / 2 - v_0 \sin \theta t - h = 0
\]

Solve for \[t\]:
\[
t = \frac{v_0 \sin \theta \pm \sqrt{v_0^2 \sin^2 \theta + (4)(g/2)(h)}}{(2)(g/2)}
\]
\[ t = \frac{v_0 \sin \Theta + \sqrt{\frac{v_0^2 \sin^2 \Theta}{g}} + 2gh}{g} \]

Take the positive solution for time.

5. \( F = ma \)
   Mass 1: \( T - F_1 = (35/g)a \)
   Mass 2: \( F_2 - T = (50/32)a \)

   Substitute for \( T \)
   \[
   F_2 - F_1 = A(35/32 + 50/32) \\
   50 - 17.5 = (85/32)a \\
   a = (32.5)(32/85) \\
   a = 12.2 \text{ ft/sec}^2
   \]
   Assume at \( t=0 \), \( V_0=0 \)
   \( v(t) = (12.2)t \)

6. Assuming no friction, the spool will spin and move to the left \( (\Sigma F_x = ma_x) \). Assuming friction, the spool will move to the left, in the direction of the unbalanced force. When moments are summed about point \( A \), the direction of the angular acceleration \( \alpha \), will be that of the moment caused by the tension of the string.

\[
\alpha = \frac{T(R-r)}{I}
\]

7. Force \( F_x \)
   \[ F_r = \sqrt{F_x^2 + \frac{2mg}{m}} \]
   \[ a_r = \frac{F_r}{m} = \sqrt{\frac{F_x^2}{m^2} + g^2} \]
   Assume \( v_0 = 0 \)
   \[ v_r = a_r t = (\sqrt{\frac{F_x^2}{m^2} + g^2}) t \]
8. The only force acting on an object in orbit is a centripetal one, gravity.

\[
F = \frac{(G m_1 m_2)}{r^2}
\]

The gravitational force causes an inward acceleration causing the velocity to change to form a circular orbit:

\[
F = \frac{(G M m E)}{r^2} = \frac{mv^2}{r}
\]

\[
\therefore \quad v = \sqrt{\frac{(GmE)}{r}}
\]
is required to maintain orbit of radius \(r\).

9. Determine position of particle as a function of time

\[
x = v_0 \cos \theta t - \frac{at^2}{2}
\]

\[
y = v_0 \sin \theta t - \frac{gt^2}{2}
\]

Determine \(x\) as a function of \(\theta\)

At impact, \(y = 0\)

\[
0 = v_0 \sin \theta t - \frac{gt^2}{2}
\]

\[
\therefore \quad t = 0, \quad 2v_0 \sin \theta g
\]

\[
\therefore \quad x = v_0 \cos \theta \left[\frac{(2v_0 \sin \theta)}{g}\right]
\]

\[
x = \frac{2v_0 \cos \theta \sin \theta}{g}
\]

\[
x = \frac{v_0^2 \sin^2 \theta}{g}
\]

Differentiate with respect to theta and you find that \(x\) will yield a maximum value when \(\theta = 45\) degrees.

10. Acceleration is the change in velocity with respect to time, or

\[
\vec{a} = \frac{d\vec{v}}{dt}
\]

11. Equate the distance required to travel.

\[
4t = 8 + \frac{1}{2}at^2
\]

\[
4t = 8 + \frac{1}{2}t^2
\]

\[
t^2 - 8t + 16 = 0
\]

\[
(t - 4)^2 = 0
\]

\[
t - 4 = 0
\]

\[
t = 4 \text{ seconds}
\]
12. I. Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it. (Equilibrium $\Sigma F=0$)

II. The change of motion is proportional to the motive force impressed; and is made in the direction of the straight line in which that force is impressed. ($\Sigma F=ma$)

III. To every action there is always opposed an equal reaction; (Action/Reaction)

13. System 1: $10 - 10 = (10/32)a$

System 2: $15 - T = (15/32)a$

Adding equations together yields

$5 = (25/32)a$

$a = 32/5$ or $6.4$ ft/sec$^2$

14. $\Theta = \omega t$ where angular velocity $\omega$

is measured in radian/sec

$x(t) = R \cos(\omega t)$

$v(t) = dx/dt = -R\sin(\omega t)$

15. Once the block is in motion the resultant forces must equal zero, thus:

1) $F\cos \Theta = F_r = N\mu_k$

2) $N + F\sin \Theta = W$

$N = W - F\sin \Theta$

$F\cos \Theta = \mu_k (W - F\sin \Theta)$

$F\cos \Theta = \mu_k (W - F\sin \Theta)$

$F (\cos \Theta + \mu_k \sin \Theta) = \mu_k W$

$F = \frac{\mu_k - W}{(\cos \Theta + \mu_k \sin \Theta)}$
16. Remember that the acceleration of the mass towards the ground is not affected by the horizontal velocity \( v_0 \).

\[
\begin{align*}
y &= gt^2/2 \\
h_1 &= gt^2/2 \\
t^2 &= 2h_1/g \\
t &= \sqrt{2h_1/g}
\end{align*}
\]

17. The second law states that the change of motion is proportional to the motive force impressed. Defining "change of motion" as "change in velocity" the following is true:

\[
\frac{F}{(d\dot{v}/dt)} = \text{constant (for a given body)}
\]

call that quantity for the given body, mass \((m)\)

\[
\frac{F}{(d\dot{v}/dt)} = m
\]

\[
F = m \cdot \dot{v}/dt
\]

\[
F = m \cdot \ddot{a}
\]

18. \( d = v_0 (\sin \Theta) t - gt^2/2 \)

Differentiate \( d \) with respect to \( \Theta \) since that is the variable of interest.

\[
d' = v_0 (\cos \Theta) t \\
0 = v_0 (\cos \Theta) t
\]

\[
\cos \Theta = 0 \quad \Theta = 90^\circ \text{ degrees or } 270^\circ \text{ degrees}
\]

\( 90^\circ \) is the angle required to maximize vertical distance travelled.

19. Inertia is the tendency of a body to remain stationary or continue in motion once started.

20. Condition to be satisfied for equilibrium to be maintained is the summation of torques about a given point is equal to zero.

\[
\sum T_a = 0 = I \alpha
\]

\[
\sum T_a = F_1(L/2)\cos \Theta + F_2(L/2)\sin \Theta - F_3(L/2)\sin \Theta + F_4(L/2)\cos \Theta + (L/2 - h)W\sin \Theta = 0
\]

At the maximum value of \( \Theta \), the frictional forces will be at their maximum value \( \mu_s N \).
\[ \Sigma T_b = 0 \]
\[ F_4 \cos \theta + F_4 \mu_s \sin \theta = (W_L/2) \sin \theta + (Wh/L) \sin \theta \]
2. \[ F_4 = \left[ (1/2)W_L + Wh/L \right] \sin \theta / (\cos \theta + \mu_s \sin \theta) \]
3. \[ F_2 = F_4 \mu_s \]
4. \[ \Sigma T_c = 0 \]
\[ F_1 \mu_s \sin \theta - F_1 \cos \theta = (W(L-h) + W_L/2) \sin \theta \]
5. \[ F_3 = F_1 \mu_s \]

Equations 2 through 5 can be substituted into equation 1. The only unknown is \( \theta \), which can be solved for using techniques of numerical analysis.

21. \( \frac{dx}{dt} = V \)
\[ \int dx = \int V \, dt \]
\[ x - x_0 = V \Delta t \] (assuming \( V \) is constant)

22. Time to fall to ground is:
\[ H = gt^2/2 \quad t = \sqrt{2H/g} \]
Distance travelled in this time assuming negligible air resistance is:
\[ x = v_0 \quad t = v_0 \sqrt{2H/g} \]

23. At the instant the block begins to slide, the frictional force will be
\[ \Sigma F_x = ma = 0 = mg \sin \theta - mg \mu_s \cos \theta \]
\[ \frac{\sin \theta}{\cos \theta} = \frac{mg \mu_s}{mg} \]
\[ \theta = \tan^{-1} \mu_s \]

24. The gravitational force between two objects is
\[ F = G(m_1m_2/r^2) \]
Therefore, if the distance \( r \) is doubled, the force is reduced to 1/4 of its original force.
25. Determine time required to reach ground.

\[ a_y = \frac{dv}{dt} \]

\[ \int a_y \, dt = \int dv \]

initial velocity in y-direction is zero.

\[ at = v - v_0 \]

\[ \int dy = \int at \, dt \]

\[ \Delta y = \frac{at^2}{2} \]

\[ t = \sqrt{\frac{2\Delta y}{a}} \]

Assume negligible air resistance

\[ \frac{dx}{dt} = v \]

\[ \Delta x = v_x \cdot t = v_x \sqrt{\frac{2\Delta y}{a}} = 10 \text{ ft/s} \sqrt{\frac{2(4 \text{ ft})}{(32.2 \text{ ft/sec}^2)}} \]

\[ = 4.98 \text{ ft} \]

26. In the y-direction (assuming negligible air resistance)

\[ v_y = v_0 \sin \theta - gt = \frac{dv}{dt} \]

\[ y = v_0 (\sin \theta) t - \frac{gt^2}{2} \]

In the x-direction

\[ v_x = v_0 \cos \theta \]

\[ x = v_0 \cos \theta t \]

The path of the projectile can be solved for by substitution

\[ y = (\tan \theta) x - \left( \frac{g}{2} v_0^2 \cos^2 \theta \right) x^2 \]
IV. FLUID DYNAMICS & BUOYANCY

1. The pressure at \( y = 0 \) will be:

\[
p = \rho g h
\]

Determine the height \( (h) \) as a function of time

\[
\frac{dh}{dt} = \frac{dv}{dt}/\frac{dv}{dn} = \frac{[(3 - t) - 2^2]}{(6 - h/2)}
\]

\[
\int (6 - h/2)dh = \int (3 - t - t^2) dt
\]

\[
6h - h^2/4 = 3t - t^2/2 - t^3/3 + C
\]

Determine \( C \). At \( t=0 \), \( h=2 \)

\[
12 - 1 = C
\]

\[
C = 11
\]

\[
h^2/4 - 6h + (11 + 3t - t^2/2 - t^3/3) = 0
\]

\[
h = 12 \pm 2\sqrt{36 - 11 - 3t + t^2/2 + t^3/3}
\]

at \( t=0 \), \( h=2 \)

\[
h = 12 \pm 2\sqrt{25}
\]

\[
2 = 12 - 2\sqrt{25}
\]

\[
\therefore h = 12 - 2\sqrt{25 - 3t + t^2/2 + t^3/3}
\]

\[
P(t) = g (12 - 2\sqrt{25 - 3t + t^2/2 + t^3/3})
\]

2. A Venturi meter utilizes a nozzle to measure the mass flow rate of a fluid by measuring the differential pressure.

---

General Energy Equation:

\[
q - w = \Delta gpe + \Delta ppe + \Delta ke
\]

\[
0 - 0 = 0 + (P_1 - P_2)/ + (v_2^2 - v_1^2)/2
\]

\[
2 \quad 2
\]

\[
(v_2 - v_1)/2 = (P_1 - P_2)/
\]

Continuity Equation

\[
\dot{m} = \rho_1 A_1 \quad \dot{m} = \rho_2 A_2
\]

\[
\dot{v}_1 = \dot{m}/A_1 \quad \dot{v}_2 = \dot{m}/a_2
\]
Substituting:
\[ \left( \frac{\dot{m}}{\rho A_2} \right)^2 - \left( \frac{\dot{m}}{\rho A_1} \right)^2 = \frac{2(P_1 - P_2)}{\rho} \]
\[ \dot{m}^2 \left[ \frac{(A_1^2 - A_2^2)}{\rho A_1 A_2} \right] = 2(P_1 - P_2) \]
\[ \dot{m} = \sqrt{\frac{2A_1^2 A_2^2}{(A_1^2 - A_2^2)}} \sqrt{\frac{P_1 - P_2}{p_1 - p_2}} = C_{\text{venturi}} \sqrt{\frac{P_1 - P_2}{p_1 - p_2}} \]

3. Archimedes Principle states that an object partly or wholly immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. While in the boat, the entire weight of the anchor is buoyed up by amount of water equal to that weight which is: \[ V_A \text{ is volume of anchor} \]

\[ \text{Volume of Water Displaced} = V_A \left( \rho_{\text{lead}} / \rho_{\text{water}} \right) = V_A \left( \frac{11.34}{1} \right) = 11.34 V_A \]

When the anchor is resting on the lake's bottom it displaces an amount of water equal to its volume.

\[ \text{Volume of Water Displaced} = V_A \]

Therefore 10.34 \( V_A \) less water would be displaced with the anchor in the water, so the level would be lower.

4. Assume that there is negligible air resistance and ideal conditions at the spout (\( C_v = 1 \))

Perform Energy Balance at spout

\[ E_p = E_k \]
\[ mgh_1 = m\frac{V_0^2}{2} \]
\[ V_0^2 = 2gh_1 \]
\[ V_0 = \sqrt{2gh_1} \]
Determine time to fall through height $h_2$

$$h_2 = \frac{\sqrt{V_0^2 - g t^2}}{g}$$

$$t = \frac{\sqrt{2h_2}}{g}$$

$$d = V_0 t + a_x t^2/2$$

$$d = (\sqrt{2gh_1}) - (\sqrt{2h_2/g}) + 0$$

$$d = 2\sqrt{h_1 h_2}$$

5. Volume of cone = $r^2 \frac{h}{3} = (6cm)^2 (10cm)/3$

$$= 377 \text{ cm}^3$$

Density of cone = $300g/377\text{cm}^3 = .8g/cc$ which is less than water which implies that the cone will float. Therefore the weight of the water displaced is equal to the weight of the cone:

$$300g = (1g/cc) (\pi) (15cm)^2 \Delta h$$

$$= 300cm/ \pi (15)^2 = .4cm$$

Use calculus to find cone volume

$$V = \int_0^h \int_0^{2\pi} r g(h - z)/h \, dr \, d\theta \, dz$$

6. Place a Pitot Tube at various radii within the pipe to determine the static pressure head, from which the velocity can be determined.

$$h = \frac{p}{\gamma} = \frac{v^2}{2g}$$

Once the velocities at several radii have been determined, they can be summed to determine the volumetric flow rate.

$$G = \sum_{i=1}^n V \pi (r_i^2 - r_{i-1}^2)$$
7. A "physically" and E ideally

Due to turbulent flow, head losses occur in the system causing a drop in pressure from pt A to pt E. Because the kinetic energy and gravitational potential energy are greater at the other points, the pressure head must be lower at those points in accordance with the conservation of energy theory.

8. Pressure is lower at pt B than pt A due to energy losses in the pipe known as head losses. This energy is converted into heat.

9. Laminar flow is a fluid flow characterized by non-turbulence and inefficient heat transfer capability. Turbulent flow has a great deal of mixing and friction. Laminar flow is more desirous in piping because pump power is reduced due to the lower head loss. Turbulent flow is desirable in situations in which heat transfer is to be maximized.

10. One pound of wood will raise the level higher. The block of wood, being less dense than water, will displace an amount of water equal to its weight. The pound of iron will only displace its own volume of water when submerged. Since the density of iron is greater than that of water, this volume represents a smaller volume than that displaced by wood.

As an example, the volumes of water displaced by 1 kg blocks of iron (\( \rho = 7.88 \text{ Kg/l} \)) and white oak (\( \rho = .68 \text{ Kg/l} \)) will be determined.

a) Volume of iron \( = \frac{m_{\text{Fe}}}{\rho_{\text{Fe}}} = \frac{1\text{Kg}}{(7.88\text{Kg/l})} = .127 \text{ liters} \)

This will be the volume of water displaced since the iron will be totally submerged.

b) Since the density of white oak is less than that of water, it will float. Archimedes’ principle states that a body wholly or partly immersed in a fluid is buoyed up by a force equal to the weight of the fluid it displaces. To remain in equilibrium, this buoyancy force must be equal to the weight of the wood (\( \sum F = ma = 0 \)). Therefore, one kilogram of water must be displaced. This water has a volume of:

\[ V = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{1\text{Kg}}{1\text{Kg/L}} = 1 \text{ Liter} \]

The wood displaces more water. Therefore, it will raise the level more.
V. WAVE PROPERTIES & OSCILLATIONS

1. When the restoring force on a mass obeys Hooke's Law \((F=kx)\), the motion of the mass is called simple harmonic motion.

\[ \Sigma F_x = ma_x \]
\[ -kx = m \frac{d^2x}{dt^2} \]
\[ m \frac{d^2x}{dt^2} = -kx \]
\[ m \frac{d^2x}{dt^2} + kx = 0 \]
\[ m^2 \omega^2 = \frac{k}{m} \]

let \( x = e^{\pm \omega t} \), take the 1st and 2nd derivatives and substitute into the eq. and get

\[ m = \pm \sqrt{\frac{k}{m}} \]

let \( x = A \cos(\sqrt{k/m} \phi + \phi_0) \) where \( A \) is the Maximum Amplitude; \( \sqrt{k/m} \) is the angular frequency \((\omega)\)

Period \( T = 2 \pi / \omega = 2 \pi \sqrt{m/k} \)

2. \( T = 2 \pi \sqrt{m/k} \) for simple harmonic motion

\[ F = -mg \sin(\theta) \sim -mg \theta \] for small angles

\[ S = L \theta \]

\[ F = -(mg/L)S \]

This equation is analogous to Hooke's Law, \( F = -kx \), where \( S = x \) and \( k = mg/L \).

\[ T = 2 \pi \sqrt{m/(mg/L)} = 2 \pi \sqrt{L/g} \]

3. Both are electromagnetic waves. The frequency of radio waves is between \( 10^3 \) and \( 10^9 \) Hz, whereas the frequency of visible light is approximately \( 10^{15} \) Hz.

4. All electromagnetic waves travel through a vacuum at the speed \( c = \lambda f \)

5. \( f \lambda = c \)
\[ f = \frac{c}{\lambda} = (3 \times 10^8 \text{ m/sec})/5 \times 10^{-10} \text{ m} \]
\[ = 6 \times 10^{17} \text{ sec}^{-1} \]
6. Light waves are transverse waves where each particle of the wave moves at right angles to the direction of travel. Its speed is affected by the index of refraction of the medium. Sound waves are compressional waves composed of regular pressure variations which move in a direction parallel to the line of travel. Speed is determined by the massiveness of vibrating particles and the elastic forces between them.

7. A Doppler Shift is an apparent change in frequency of a wave due to motion of the source in reference to the receiver.

8. Radiation

<table>
<thead>
<tr>
<th></th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFRARED</td>
<td>$10^{11} - 7 \times 10^{14}$</td>
</tr>
<tr>
<td>VISIBLE LIGHT</td>
<td>$7 \times 10^{14} - 2 \times 10^{15}$</td>
</tr>
<tr>
<td>X-RAYS</td>
<td>$5 \times 10^{16} - 5 \times 10^{21}$</td>
</tr>
<tr>
<td>Y-RAYS</td>
<td>$3 \times 10^{18} - 5 \times 10^{22}$</td>
</tr>
</tbody>
</table>

9. A light beam at angle $\theta_1$ in material 1 makes an angle in material 2 such that: $u_1 \sin(\theta_1) = u_2 \sin(\theta_2)$

where: the quantities $u_1$ and $u_2$ are the two refractive indices (ratio of the speed of light in a vacuum to the speed of light in the material)

10. The spear should be shot closer to you than where the fish appears to be due to refraction. The light waves from the fish increase in velocity upon reaching the less dense medium (air), and bend towards the horizontal. C, the speed of light in free space, provides the reference for other velocities and is in Snell's Law: $n_a \sin(a) = n_b \sin(b)$ where:

$n = \frac{C}{v}$ (speed of light in a vacuum) is the index of refraction

$v$ (speed of light in material)
11. In both cases, light is "bent" toward the normal upon entering the glass and "bent" away from the normal upon leaving the glass all due to laws of refraction. The net result is to bend the light to one point for the convex lens and away from one point for the concave lens.

convex

concave

12. 1) It causes light to interfere with itself. The interference pattern is a function of wave-length, angle between pattern and grating and distance between grating elements. I.e. $\sin = m / d$, where $m=0,1,2,3,...,n$. The grating is used in spectroscopy as a means of dispersing a light beam into spectra.

2) Yes. Photo-electric effect, Compton scattering, shadows.
VI. ELECTROMAGNETICS

1. Capacitance (C) is the measure of the number of coulombs of charge (Q) the capacitor holds for each volt of potential difference (V).

\[ C = \frac{Q}{V} \]

The induced EMF (E) across an inductor is proportional to the change in current with respect to time (\( \frac{di}{dt} \)). Inductance (L) is the constant of proportionality and depends upon the geometry of the coil.

\[ E = -L \frac{di}{dt} \]

2. The voltage across a parallel plate capacitor in a vacuum is \( V = Ed \) where \( E \) is the electric field and \( d \) is the separation of the plates, so \( V=EL \) and

\[ E = \frac{V}{L} \]

The force on a charged particle in an electric field is defined as \( F = Eq \).

Also \( F=ma \) so \( ma = Eq \) where \( a \) is the acceleration of the particle.

Therefore \( a = \frac{Eq}{m} = \frac{Vq}{Lm} \). Since \( L = \frac{at^2}{2} \), \( t^2 = \frac{2L}{a} \) where \( t \) is the time for the particle to travel from plate to plate. Also \( v = at \) where \( v \) is the velocity of the particle.

So \( t = \frac{v}{a} \). Eliminating \( t \) from the two equation yields

\[ \frac{v^2}{a} = \frac{2L}{a} \]

Solving for \( v \) gives the results \( v = \frac{2Vq}{m} \).

3. With every current through a wire, there is a corresponding magnetic field around the wire. The magnetic field is related to the current by the equation

\[ B = \frac{kI}{r} \]

where \( K \) is a constant of proportionality and \( r \) is the distance from the wire. Therefore measuring the strength of the magnetic field at a certain distance from the wire will allow the current to be determined.

The strength of the magnetic field can be easily measured for a changing current because the changing magnetic field associated with a changing current will induce a current in a coil next to the wire. Measuring the current through the coil will effectively provide a means of determining the current in the wire without touching it.

Steel or iron ring to concentrate the mag. field

Meter calibrated to read mag. fields

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4. The current through $R_3$ can be determined using equivalent resistances.

Adding $R_2$ and $R_3$ in parallel results in $R_{2,3} = \frac{R_2 R_3}{R_2 + R_3}$

Using Kirchhoff's voltage law, $V = R_1 I_t + R_{2,3} I_t = R_1 + \frac{R_2 R_3}{R_2 + R_3} I_t$

Hence, $I_t = V R_1 + \frac{R_2 R_3}{R_2 + R_3} \quad V_{R_{2,3}} = I_t R_{2,3} = V \left(1 + \frac{R_1 (R_2 + R_3)}{R_2 R_3}\right)^{-1}$

$V_{R_{2,3}} = \frac{V_{R_3}}{R_3} \quad I_3 = \frac{V_{R_{2,3}}}{R_3}$

$I_3 = \frac{V}{R_3} \left(1 + \frac{R_1 (R_2 + R_3)}{R_2 R_3}\right)^{-1}$

5. Assume: 1) switch is close at time $= 0$
   2) there is no initial charge on the capacitor $q(0) = 0$.

By definition $I(t) = \frac{dq}{dt}$ charge passing a point in a wire

Using Kirchhoff's voltage law $E = V_r + V_c$

Also by definition $C = \frac{q}{V_c}$ charge on the plates of a capacitor

voltage across the capacitor
so \( V_c = \frac{q}{C} \)

and \( V_r = RI = \frac{dq}{dt} \)

\[ V_r + V_c - E = 0 = R \frac{dq}{dt} + \frac{1}{C} q - E \]

Dividing by R yields \( \frac{dq}{dt} + \frac{1}{RC} q - \frac{E}{R} = 0 \)

Solving the differential equation results in \( q(t) = EC(1 - e^{-t/RC}) \)

Since \( I(t) = \frac{dq}{dt} \), \( I(t) = \frac{EC}{RC} e^{-t/RC} \)

\[ I(t) = \frac{E}{R} e^{-t/RC} \]

\[ V_r = I(t)R = E e^{-t/RC} \]

\[ V_c = \frac{1}{C} q = E(1 - e^{-t/RC}) \]

The following diagrams show these currents and voltages graphically.

Energy dissipated by the resistor:

power = \( P = IV \)

energy = \( W = \int P \, dt = \int IV \, dt \)

substituting in for \( I(t) \) and \( V_r \):

\[ W = \int_0^\infty \left( \frac{E}{R} e^{-t/RC} \right) E (e^{-t/RC}) \, dt \]

\[ W = \frac{E^2}{R} \int_0^\infty e^{-2t/RC} \, dt = \frac{CE^2}{2} \]

Energy stored in the capacitor:

energy = \( W = IV_c \, dt \)

substituting in for \( I(t) \) and \( V_c \):

\[ W = \int_0^\infty \left( \frac{E}{R} e^{-t/RC} \right) E (1 - e^{-t/RC}) \, dt \]

\[ W = \frac{E^2}{R} \int_0^\infty \left( e^{-t/RC} - e^{-2t/RC} \right) \, dt = \frac{E^2C}{2} \]

The energy supplied by the battery is the sum of the energy stored in the capacitor and the energy dissipated by the resistor therefore the energy contributed by the battery is

\[ W = E^2C \]

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so \( V_C = \frac{q}{C} \) and \( V_r = RI = R \frac{dq}{dt} \)

\[ V_r + V_C - E = 0 = R \frac{dq}{dt} + \frac{1}{C} q - E \]

Dividing by \( R \) yields \( \frac{dq}{dt} + \frac{1}{RC} q - \frac{E}{R} = 0 \)

Solving the differential equation results in \( q(t) = EC(1 - e^{-t/RC}) \)

Since \( I(t) = \frac{dq}{dt} \), \( I(t) = \frac{EC}{RC} e^{-t/RC} \)

\[ I(t) = \frac{E}{R} e^{-t/RC} \]

\[ V_r = I(t)R = E e^{-t/RC} \]

\[ V_C = \frac{1}{C} q = E(1 - e^{-t/RC}) \]

The following diagrams show these currents and voltages graphically.

Energy dissipated by the resistor:

\[ P = IV \]

\[ W = \int P \, dt = \int IV \, dt \]

substituting in for \( I(t) \) and \( V_r \):

\[ W = \int_{0}^{\infty} \left( \frac{E}{R} \right) e^{-t/RC} E \left( e^{-t/RC} \right) \, dt \]

\[ W = \frac{E^2}{R} \left[ -2E \right]_{0}^{\infty} = \frac{CE^2}{2} \]

Energy stored in the capacitor:

\[ W = IV_C \, dt \]

substituting in for \( I(t) \) and \( V_C \):

\[ W = \int_{0}^{\infty} \left( \frac{E}{R} \right) e^{-t/RC} E (1 - e^{-t/RC}) \, dt \]

\[ W = \frac{E^2}{R} \left[ e^{-t/RC} - e^{-2t/RC} \right]_{0}^{\infty} = \frac{E^2C}{2} \]

The energy supplied by the battery is the sum of the energy stored in the capacitor and the energy dissipated by the resistor therefore the energy contributed by the battery is

\[ W = E^2C \]
6. Assume the batteries are all of the same voltage $V$. Summing the voltages in series yields $3V$ volts between points $A$ and $B$. Since $v = \frac{i}{R}$, $3V = I_rR$

\[ I_r = \frac{3V}{R} \]

7. To accurately graph $I_1$ and $I_2$, the mathematical equation must first be found.

The voltage across $R$ is $V$ so $V = I_1R$. \[ I_1 = \frac{V}{R} \]

The voltage across $L$ is also $V$ so $V = \frac{dI_2}{dt} - L$ so $V \, dt = dI_2$. \[ I_2 = \frac{Vt}{L} \]

A graph of these functions is shown below.

This circuit is unrealistic because there is no resistance in series with the inductor to limit $I_2$. 

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8. Arithmetically the \( V_{av} = V(\int_{0}^{\pi} \sin(\theta))/3\pi = (V)(2)/3\pi = 2V/3\pi \) which is what a dc voltmeter would indicate. But for sinusoidal voltage, the "average" or "effective" value is \( V/\sqrt{2} = V_{rms} \)

9. Introduce an electron beam into a perpendicular magnetic field. Vary the magnetic field until the beam reverses direction forming a half circle. Measure the radius and calculate.

\[
F_B = qvB \\
F_C = mv^2/r \\
F_B = F_C \text{ therefore } qvB = mv^2/r \\
v = rqB/m \\
F_B = qvB = rq^2B^2/m
\]

10. **Advantages**

- Have less weight and take up less space than 3 single phase
- More economical per kilowatt
- Transmission lines cheaper
- More reliable

11. **Disadvantage**

Larger Generators

11. **P = I V**

300 watts = \( P(225 \text{ volts}) \)

\[
I = \frac{300}{225} \text{ amps} = 1.34 \text{ amps}
\]

12. Resistance is the ratio of voltage to current in a DC circuit whereas impedance is the quotient of the vector of voltage and the vector of current in an AC circuit. Impedance is composed of an resistance term and a reactance term.

13. Current will take a prompt jump when the switch is shut due to charging the capacitor. The current through the resistor remains constant since the potential is constant. The current of the circuit will decrease exponentially to its original value in five RC time constants.
14. Maxwell's equations contain all the information necessary to characterize the electromagnetic field, its generation, its propagation through space and its absorption. They are:

\[ \nabla \cdot \vec{E} = \frac{P}{\varepsilon_0} \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} + \mu_0 \vec{J}_c \]
\[ \nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \]
\[ \nabla \cdot \vec{J}_c = -\frac{dP}{dt} \]

15. Yes, the earth's magnetic field can be simulated, using a coil of wire.

16. Faraday observed that if the same electric charge is transversed through different electrolytes, the masses of the deposited materials on the electrodes are proportional to the chemical equivalent weights of the material. This implied atomicity of matter. He additionally developed a law for induction:

\[ \varepsilon = -N \frac{\Delta \Phi}{\Delta t} \]

where: \( N \) is the number of loops of conductor
\( \Phi \) is the magnetic flux through the coil

17. Yes - at the north or south magnetic poles due to the magnetic field being more concentrated.
19. \( \frac{1}{10} + \frac{1}{20} + \frac{1}{R} = \frac{1}{6} \)
   \[ \frac{3}{20} + \frac{1}{R} = \frac{1}{6} \]
   \[ \frac{1}{R} = \frac{10}{60} - \frac{9}{60} \]
   \[ \frac{1}{R} = \frac{1}{60} \]
   \[ R = 60 \, \text{ohms} \]

20. Figure 1

\[ V = IR \quad \therefore I = \frac{V}{R} \]

Figure 2

Using admittances (Y)
\[ Y_R = \frac{1}{R} \quad Y_L = -\frac{1}{j\omega C} \]

\[ I = I_R + I_L = VY_R + VY_L \]

\[ \therefore I = V\left(\frac{1}{R} - 1/j\omega L\right) \quad \text{and} \quad I = V\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2 \]
\[ Y_C = jwC \]
\[ I = I_R + I_L = V(1/R + jwC) \]
\[ \therefore \quad I = V \left(1/R\right) + (wC)^2 \]
\[ \therefore \quad I_{F_1} < I_{F_2} \]
\[ \text{and } I_{F_1} < I_{F_3} \]

For zero capacitance \[ |I| = V/R \]

For Infinite Capacitance, the current would only be limited by the current capabilities of the voltage supply or the wire resistance.

21. Magnetic fields exert a force on wires carrying current. The force is equal to \[ F = I L B \] where \( I \) = current in the wire, \( L \) = length of wire exposed to magnetic field and \( B \) = strength of magnetic field.

If a loop of wire with a current passing through it is immersed in a magnetic field of length \( a \) and width \( b \), the sides of the coil will have a equal but opposite force of \[ F = IaB \] at a distance \( b/2 \) from the axis of the loop. This will cause the loop to have a torque of:

\[ \tau = 2F \cdot R = Fb \cos(\theta)/2 \]

where \( F = a I B \)

\[ = a I b \cos(\theta)/2 \]

This torque will cause the loop to rotate until the plane of the loop is perpendicular to the magnetic field lines (\( \theta = 90^\circ \)). At this point, if the current is reversed, the forces will reverse and the loop will continue to rotate in the same direction due to the inertia of the loop carrying it through the \( \theta = 90^\circ \) position.
22. Measure physically
Use Kirchoff's and Ohm's Laws to show that you can find an equivalent resistance for a group of resistors.

For example - series resistors

\[
\begin{array}{c}
R_1 \\
+ V_1 \\
\hline
R_2 \\
+ V_2 \\
\hline
R_3 \\
+ V_3 \\
\hline
V_T \end{array}
\]

From Kirchoff's loop law

\[ V_T = V_1 + V_2 + V_3 \]

From Ohm's law

\[ V_1 = IR_1 \quad V_2 = IR_2 \quad V_3 = IR_3 \quad V_T = IR_{eq} \]

\[ \therefore \quad IR_{eq} = IR_1 + IR_2 + IR_3 \]

So if a single resistor \( R_{eq} \) (\( R_{eq} = R_1 + R_2 + R_3 \)) replaces \( R_1 \), \( R_2 \), and \( R_3 \) the circuit would still have the same voltage and currents.
VII. ATOMIC

1. X-ray energy levels are continuous due to the production process of Bremsstrahlung where a high energy passes near a massive nucleus and creates a photon as it slows. \( E_{k_1} - E_{k_2} = h \omega \). The frequency distribution does have some high peaks at discrete frequencies where a lower level electron was shifted to an outer shell following a collision and the unstable atom releases a photon when another electron fills the vacancy.

2. The time-independent Schrödinger wave equation is:

\[
\frac{\hbar^2 \nabla^2 \psi}{2m} = (\nu - E)\psi
\]

where \( \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \)

is the spatially-dependent wave function, \( \nu \) is potential energy, and \( E \) is total energy.

The Schrödinger equation can be used to extract the probability of location and energy and momentum of a system. It governs the wave mechanics for matter.

If the potential energy function of a bound particle, such as Hydrogen, is known then the allowed wave functions and energies may be found.

3. Bohr postulated that an atom's electrons can exist in only certain stable, discrete orbits. Each state corresponds to a certain energy. When the electron is infinitely far from the atom, the energy of this state is defined to be zero. The ground state of the atom has the lowest possible energy. Unexcited atoms are in the ground state. When the electron falls from one level to a lower one, the atom changes from one state to another. The emitted photon has energy equal to the differences in energy between the two states. The photon energy is related to the wavelength of the emitted spectral line:

\[
\text{Photon energy} = \frac{hc}{\lambda}
\]

4. Contain it in a "magnetic bottle" by using magnetic fields to hold the substance in the vacuum.

5. \( N_L = N_0(1 - e^{-at}) = N_0(1 - e^{-[.693/T(1/2)](t)}) \)

\[
\frac{N_L}{N_0} = 1 - e^{-[.693/40 \text{ yr}](60 \text{ yr})}
= 1 - e^{-1.04}
= 1 - .35
\]

\[
\frac{N_L}{N_0} = .65
\]
1. Tempering is a stress relief anneal which will cause defects in the glass to diffuse through the material rather than being concentrated at particular locations. This increases the ductility of the glass and allows it to absorb more plastic deformation before failing and therefore is harder to break.

2. Young's modulus is the ratio of tensile stress to tensile strain of a particular material.

\[ Y = \frac{\sigma}{\varepsilon} \]

Young's modulus for steel is \( 2.0 \times 10^{12} \) dynes/cm or \( 2.9 \times 10^7 \) lb/in²

3. Castings
   Good for intricate shapes
   Finish is rough
   Tolerances are poor

Forgings
   Excellent strength
   High toughness
   Excellent Tolerances
   Shapes are limited

4. Shear and Flexural Stress

\[ \text{Shear} = \frac{VQ - (\text{shear load at point}) (I_b \text{ moment about pt of desired shear})}{(I_b \text{ moment of inertia}) (\text{distance to point of desired shear})} \]

\[ \text{Flexural stress} = \frac{Mc}{I_e} \]

5. [Shear and Moment Diagrams]

6. Hooke's Law - If a material is elastic then stress is directly proportional to strain.

\[ (\text{Newtons/m}^2) = K (\Delta l/l_0) \]

Poisson's Ratio is the ratio of the transverse contraction per unit dimension of a bar of uniform cross-section to its elongation per unit length,
when subjected to a tensile stress. Similarly, amount of "necking" divided by amount of "stretching".

7. "Cold Working" is a process by which the strength of a material is increased by causing physical stress at below recrystallization temperature. This work hardening causes dislocations in the crystal structure to cancel or "pile up" at the grain boundary. The end result is a stronger material that is able to resist penetration or abrasion (hardness). During an annealing process, the material is heated and cooled to reduce brittleness (increase ductility). The heating allows crystal defects to diffuse through the material. The cooling allows new grains to grow. The effect reduces the concentration of defects in the material allowing more plastic deformation.

8. \[ W = \int F \cdot dr = \left( \sigma_y A_0 \right) d(L_0 \varepsilon) \]

Work is the area under the curve times the volume of the specimen.

9.

![Shear and Moment Diagrams](image-url)
1. Diodes are constructed of semiconductor materials such as silicon and germanium. In their pure crystalline form, these elements have a very high resistance. When small a amount of a material, which has a different number of valence electrons than the semiconductor element, is added to the semiconductor material, the resulting material has a high conductivity. If the added material has more valence electrons than the semiconductor element, the resulting material is called n-type material and the excess free electrons allow the conduction of current. P-type material results when a substance with fewer valence electrons is added. Current in p-type material is caused by electrons moving from one 'hole' to the next in the crystalline structure of the material. The term 'hole' describes a positive charge in the electronic structure of the crystal created from the deficiency of electrons introduced by the added material.

Diodes are constructed by making one end of a semiconductor material n-type and the other end of the material p-type as shown in the diagram. At the border between the n and p-type materials, electrons from the n-type material migrate to the p-type material to fill in some of the holes thus creating holes in the n-type material. Due to the positive charges (holes) created in the

\[
\text{n} \quad \text{p}
\]

deployment region

n-type material and the negative charges (electrons) moving to the p-type material, an electrostatic force is set up and the process will come to an equilibrium. The area where there is a reduced number of electrons in the n-type side and a reduced number of holes in the p-type side is called the deployment region.

If a positive voltage is applied to the p-type side of the diode with respect to the n-type side of the diode (forward biased) as in the diagram, electrons

\[
\text{+} \quad \text{n} \quad \text{p}
\]

forward biased

will be supplied to the n-type side and holes will be created in the p-type side. This will support the migration of charges across the depletion region and current will flow. If a negative voltage is applied to the p-type side with respect to the n-type side (reverse biased), very little current will flow

\[
\text{+} \quad \text{n} \quad \text{p}
\]

reverse biased

because the holes created in the n-type side and the electrons added to the p-type side only increase the depletion region and do not supply charges which could traverse the depletion region.

Therefore the main characteristic of a diode is that it will conduct current only in one direction, when it is forward biased.
2. The main property of a diode is that it conducts electricity in only one direction. When forward biased, a slight voltage (.5 to .8 volts) is needed to 'turn on' the diode. This voltage is necessary to overcome the internal electrostatic force set up across the depletion region. The diode will conduct with very little resistance once the applied voltage rises above the 'turn on' voltage but there will always be a voltage drop equal to the 'turn on' voltage across the diode. There is a very small reverse current when a diode is reverse biased, but it is negligible for most practical purposes. Diodes also have a characteristic breakdown voltage, at which they will conduct while reverse biased. Large currents can flow once the breakdown voltage has been reached and care must be taken to limit the current to avoid damaging the device. The current vs. time diagram for a diode is shown below.
3. Transistors are usually constructed from germanium or silicon. The electron holes for p-type material and the excess electrons for n-type material are introduced by adding impurity elements (i.e. Al or P) which have different numbers of valence electrons than the base material. These elements (dopants) may be added before the initial crystalization of the Si or Ge or they may be diffused into the surface of a wafer of the material by exposing the wafer to a gas containing the desired dopant while it is being heated in a furnace. The wafers can be masked to achieve desired patterns of alternating p and n-type layers resulting in the ability to form many transistors on one wafer.

An n-p-n transistor is made up of a thin layer of p-type material placed between two layers of n-type material. One of the n-type ends is called the emitter, the other end is called the collector and the thin middle layer is called the base. When the transistor is biased as shown in the diagram, the base-emitter junction is forward biased while the base-collector junction is reverse biased. The forward biased junction allows electrons to flow from the emitter to the base. Once the electrons enter the base, few combine with the holes which exist in the p-type base due its small size. While some of the electrons will flow out of the base terminal, most will continue through the base and into the collector and to the positive voltage supply because the base is thin and also because the positive voltage on the collector is large compared to the base voltage. The current which exits the collector is some fraction (α) of the current which enters the emitter. This fraction is determined by the construction of the transistor and is usually in the range of 0.95 to 0.99. Using Kirchhoff's current law, it is obvious that the fraction of current which does not exit the collector must exit the base, therefore:  

$$I_b = I_e - I_c$$

Since $$I_c = \alpha I_e$$, then $$I_b = I_e (1-\alpha)$$. Thus  

$$\frac{I_c}{I_b} = \alpha$$  

This relationship indicates that the collector current can be from approximately 20 to 200 times greater than the base current. It also implies that the base current can be used to control the collector current. These features make transistors useful for amplification purposes. If a small varying voltage is applied to the base in such a way as to vary the base current, then the collector current will also be varied by the amount of the base current variation times the current gain of the transistor. This effectively amplifies the signal applied to the base.

Transistors can also be used as current controlled switches. If the base-emitter junction is unbiased or reverse biased, no collector current can flow, thus the switch is 'off'. When the base-emitter junction becomes forward biased and current flows through the base, current is allowed to flow through the collector and the switch is 'on'.
4. Digital circuits are a means of representing binary numbers by different voltage levels. A high voltage level (2.5 to 5 volts) is the electric equivalent to a one and a zero is represented by a low voltage level (0 to 1 volts). Diodes and transistors are arranged in circuits which will perform a given function on a number of inputs and output a desired result. The functions are achieved by switching transistors on and off. Since the voltage levels in the circuit represent binary values, circuits can be made to perform any given math or logical function.

5. Use Kirchhoff's voltage law on the circuit below:

\[ E = V_r + V_c + V_l = IR + \frac{1}{C}q + L \frac{dI}{dt} \]

by definition \( I = \frac{dq}{dt} \) \( \therefore q = \int I \, dt \)

so \( E = IR + \frac{1}{C} \int I \, dt + L \frac{dI}{dt} \)

differentiating the equation with respect to \( t \) and dividing by \( L \) yields

\[ 0 = \frac{d^2I}{dt^2} + \frac{R}{L} \frac{dI}{dt} + \frac{1}{CL} I \]

This differential equation can be solved by assuming a solution of the form \( I = e^{mt} \). Substituting this into the equation results in the characteristic equation \( m^2 + \frac{R}{L} m + \frac{1}{CL} = 0 \). \( \therefore m = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}} \)

to simplify the analysis let \( b = \frac{R}{2L} \) and \( a = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}} \)

Therefore the general for the current is:

\[ I(t) = K_1 e^{(-b+a)t} + K_2 e^{(-b-a)t} \]

This solution has three forms.

a.) If \( \left(\frac{R}{2L}\right)^2 > \frac{1}{CL} \) then \( a \) is real and the solution is

\[ I(t) = K_1 e^{(-b+a)t} + K_2 e^{(-b-a)t} \]
since \( I(0) = 0 \), \( K_1 = -K_2 \)

\[ \therefore I(t) = K_1 e^{-bt} (e^{at} - e^{-at}) \]

\( K_1 \) can be found by substituting \( I(t) \) into the original circuit equation. Using graphing techniques, it can be shown that the graph of \( I(t) \) vs. \( t \) is of the form shown below.

![Graph of I(t) vs. t]

b.) If \( \left( \frac{R}{2L} \right)^2 < \frac{1}{CL} \) then \( a = 0 \) and the solution takes the form

\[ I(t) = K_1 e^{-bt} + K_2 t e^{-bt} \]

since \( I(0) = 0 \), \( K_1 = 0 \) and \( I(t) = K_2 t e^{-bt} \).

Again using graphing techniques, \( I(t) \) vs. \( t \) can be found as

![Graph of I(t) vs. t]

c.) If \( \left( \frac{R}{2L} \right)^2 > \frac{1}{CL} \) then \( a \) is imaginary and the solution is

\[ I(t) = K_1 e^{(-b+ia)t} + K_2 e^{(-b-ia)t} \]

using Euler’s relationship \( e^{ix} = \cos x + i \sin x \) and \( I(0) = 0 \)

\[ I(t) = K_3 e^{-bt} \sin at \]

This is an exponentially decaying sinusoid which looks like

![Graph of I(t) vs. t]

The current in cases a. and b. starts at zero and increases slowly because the inductor resists a change in current, but as time goes on, the capacitor charges and the current will eventually decay to zero.
6. The Thevenin equivalent of a circuit is used to reduce the circuit to a single voltage source and a series resistance. The magnitude of the voltage source is found by determining the open circuit voltage at the terminals of the circuit. The series resistance is found by replacing the voltage source by a short circuit and finding the equivalent resistance of the circuit at the terminals. Here is an example.

\[ E' = E \frac{R_2}{R_1 + R_2} \]

\[ R' = \frac{R_3 R_2}{R_1 + R_2} \]

7. Kirchhoff's point rule states that at any point in a circuit, the sum of the currents entering that point must equal the sum of the currents leaving that point.

Kirchhoff's loop rule states that the sum of the potential changes around any closed circuit equals zero.

8. Impedance is the ratio of voltage to current associated with a given component in an ac circuit with sinusoidal currents and voltages.

\[ Z = \frac{V(t)}{I(t)} \]

It is represented by a real part which is the purely resistive component and an imaginary part which represents the reactive or energy storage component (inductors and capacitors).

For a resistor, the impedance is \( Z_R = R \)

For a capacitor, the impedance is \( Z_C = -\frac{1}{j \omega C} \)

For an inductor, the impedance is \( Z_L = j \omega L \)

\( \omega \) indicates the frequency of the sinusoidal current or voltage in radians.

\[ Z_T = Z_R + Z_C + Z_L \]

\[ Z_T = R + \frac{1}{j \omega C} + j \omega L = R + \frac{1}{j \omega C} \]

\[ \therefore Z_T = R - \frac{1}{j \omega C} + j \omega L = R + j(\omega L - \frac{1}{j \omega C}) \]
and in polar notation:

\[ Z = \sqrt{R^2 + (wL - \frac{1}{wC})^2} \quad \angle \tan^{-1} \left( \frac{wL - \frac{1}{wC}}{R} \right) \]

9. To find the current, assume a dc voltage source and the switch is closed at time 0 so \( i(0) = 0 \).

![Diagram of circuit with voltage source, inductor, and resistor](image)

The voltage across an inductor is \( v_L = L \frac{di}{dt} \)

The voltage across a resistor is \( v_R = iR \)

Using Kirchoff's loop rule \( V = v_R + v_L = iR + L \frac{di}{dt} \)

Conventional methods can be used to solve this differential equation with the initial condition \( i(0) = 0 \).

The solution is:

\[ i(t) = \frac{V}{R} \left( 1 - e^{-\frac{Rt}{L}} \right) \]

The voltage across the inductor is \( v_L = L \frac{di}{dt} = L \frac{V}{R} \left( \frac{R}{L} e^{-\frac{Rt}{L}} \right) \)

\[ \therefore v_L = V e^{-\frac{Rt}{L}} \]

The power consumed by the inductor is \( P = iv = \frac{V^2}{R} \left( e^{-\frac{Rt}{L}} - e^{-2\frac{Rt}{L}} \right) \)

The energy stored in the inductor is:

\[ W = \int P \, dt = \int iv \, dt = \frac{V^2}{R} \int \left( e^{-\frac{Rt}{L}} - e^{-2\frac{Rt}{L}} \right) \, dt \]

\[ \therefore W = \frac{LV^2}{2R} = \text{energy stored in the inductor} \]

10. Use Kirchhoff's loop rule to find the currents.

![Diagrams of two loops with currents and voltages](image)

Loop 1: \( V = R \, i_1 + L \frac{di_1}{dt} \)

solving the differential equation yields

\[ i_1 = \frac{V}{R} \left( 1 - e^{-\frac{Rt}{L}} \right) \]
Loop2 : $V = R I_2 + \frac{1}{C} \int I_2 \, dt$

solving this differential equation yields

$$I_2 = \frac{V}{R} e^{-t/RC}$$

11. A DC motor needs a commutator to switch the direction of the current going through the coils in the armature. The direction of the current must be switched in order to reverse the forces on the wire loops in the coils, which will sustain the torque on the armature as it rotates.

An AC motor only needs slip rings because the current already alternates directions so mechanical means of switching the direction of the current is not needed.

12. An induction motor has three phase ac current supplied to the stator windings which produce a rotating magnetic field. The rotor consists of closed conductive loops with current limiting resistors in the loops. The rotating magnetic field cuts through the loops and induces a current which in turn produces a torque on the rotor. This torque will pull the rotor in the direction of the rotating magnetic field.

The stator windings on a synchronous motor are similar to those on an induction motor. The rotor has dc current applied to it in order to produce a fixed magnetic polarity on the rotor. The field set up by the rotor rotates with the stator field, thus turning the rotor synchronously with the stator field.

13. The speed of a separately excited DC motor can be increased by decreasing the field winding resistance, thus increasing the intensity of the magnetic field. The reverse emf is the emf produced by the wires of the armature passing through the magnetic field. The reverse emf is shown by $V_g$.

14. A transformer consists of 2 coils of wire wrapped around the same iron ring (core) as shown in the figure.
The varying input voltage $V_{in}$ will produce a magnetic field in the primary winding (coil) such that $V_1 = K N_1 \frac{d \Phi}{dt}$ where $K = \text{constant of proportionality}$, $N_1 = \text{number of primary windings}$ and $\frac{d \Phi}{dt} = \text{change in magnetic flux}$.

Since the secondary winding has the same flux going through it because it is wound on the same iron core, an emf is induced in it such that

$$V_2 = K N_2 \frac{d \Phi}{dt} \quad \therefore \quad \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

There are winding losses due to the resistance of the wire and the inductance of the coils. Also there are core power losses due to hysteresis and the fact that eddy currents may be induced in the core.

15.

$$I(t) \text{ can be found by using admittances in the circuit.}$$

resistive admittance $= Y_r = \frac{1}{R} \quad \therefore I_r = V Y_r = \frac{V}{R}$

capacitive admittance $= Y_C = j \omega C \quad \therefore I_C = V Y_C = V j \omega C$

I_t = I_r + I_c = V \left( \frac{1}{R} + j \omega C \right)$

converting to polar representation, $I_t = V \sqrt{\left( \frac{1}{R} \right)^2 + (\omega C)^2} \angle \tan^{-1} \frac{\omega C}{\frac{1}{R}}$

adding the time function yields

$$i(t) = V \sqrt{\left( \frac{1}{R} \right)^2 + (\omega C)^2} \sin(\omega t + \tan^{-1} \frac{\omega C}{\frac{1}{R}})$$

This shows that the current is a sinusoid which leads the voltage by an angle $\tan^{-1} \frac{\omega C}{\frac{1}{R}}$
16. As shown in the diagram below, when the input has the polarity shown, diodes 1 and 4 are forward biased, thus they conduct current and allow current to flow through the load in the direction shown. When the input polarity switches, diodes 2 and 3 become forward biased and they conduct current through the load in the same direction. A capacitor in parallel with the bridge output will filter the output to reduce the oscillation in the DC current. Maximum current is lost in this circuit.

17. The input to this circuit is a DC voltage source.

This circuit will oscillate if \( \left( \frac{R}{2L} \right)^2 < \frac{1}{LC} \) and the current will have the form \( I_{\text{max}} e^{-Rt/2L} \sin wt \) where \( w = \sqrt{\frac{1}{LC} - \left( \frac{R}{2L} \right)^2} \).

Therefore the voltage across the resistor will have the same form as the current and it can be used as the output. The output waveform is shown in the diagram and it has a frequency of \( f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left( \frac{R}{2L} \right)^2} \).

18. a. \( R_1 = 0.5 \) \( R_2 = 2R_3 \)
\[ I_L = I_1 - I_2 \quad I_1 = \frac{V}{R_1} \]
\[ I_2 = \frac{V}{R_2} = \frac{V}{2R_1} \quad \therefore \quad I_L = \frac{V}{R_1} - \frac{V}{2R_1} = \frac{V}{2R_1} \]

b. \( R_1 = R_2 = R_3 \)

The magnitude of \( I_L = \sqrt{3} \frac{V}{R_1} \)

19. A magnetic field is established with a magnetic flux density of \( B \). An external prime mover turns a coil of conducting material perpendicular to the magnetic field. The magnetic field lines which are cut by the coil induce an electromotive force in the coil. This emf is the output voltage of the generator.

\[ \mathcal{E} = -N \frac{d\phi}{dt} \quad \phi = \text{magnetic flux} = BA \cos(\alpha) \quad B = \text{fld flux density} \]

\[ \mathcal{E} = NBA \sin(\alpha) \frac{d\alpha}{dt} \quad N = \# \text{ of turns} \quad A = \text{area of coil} \]

\[ \mathcal{E} = NBA \omega \sin(\alpha) \quad \mathcal{E} = \text{induced EMF} \quad \alpha = \text{angle between B field and coil normal} \]

20. As frequency increases, reactance decreases, impedance decreases. As frequency increases, current increases.

Use admittances to find the current.

\[ Y_r = \frac{1}{R} \quad Y_C = j\omega C \]

\[ I_r = V Y_r \quad I_C = V Y_C \]

\[ I_T = I_r + I_C = V (Y_r + Y_C) = V \left( \frac{1}{R} + j\omega C \right) \]

\[ |I_T| = V \sqrt{\left( \frac{1}{R} \right)^2 + (\omega C)^2} \]

\( I_T \) is a minimum when \( \left( \frac{1}{R} \right)^2 + (\omega C)^2 \) is a minimum. To find \( \omega \) for when the equation is a minimum, differentiate with respect to \( \omega \), set the result to zero and solve for \( \omega \).

\[ 2\omega C^2 = 0 \quad \text{when } \omega = 0. \]

Obviously this minimizes \( I_T \). \( \therefore \) \( w=0, f=0 \) gives the minimum current.
21. Current decreases in a DC motor during startup due to the apparent increasing armature resistance created from the back emf. To protect the machine due to the low armature resistance upon startup, resistors are added in series to the armature. These resistors are cut out as the machine speeds up until the motor is operating at its design speed.

22. Semiconductors occur in N-type (Arsenic doped) and P-type (Boron doped) varieties. The N-type has an excess of electrons which are not in bonds due to the crystalline structure and the four covalent bonds per atom. This doping raises the level (donor) near the conduction band (.01 ev) allowing a small amount of biasing to cause electron flow. Similarly a P-type semiconductor has an excess of holes which with biasing will cause hole flow (actually valence electrons shifting due to only .01 ev required to shift it).

\[
\text{Conduction Band} \quad \frac{1}{.01 \text{ ev}} \quad \text{Donor Level} \quad \quad \text{P-type} \quad \frac{1}{.01 \text{ ev}} \quad \text{Accepton Level}
\]

23. Since all the components are in series, the current through each is that put out by the current generator.

The amplitudes of each voltage are:
\[ V_C = \mathbf{I} \times X_C = I \ 0^\circ \times 1/\omega \angle -90^\circ \]
\[ V_L = \mathbf{I} \times X_L = I \ 0^\circ \times \omega \angle 90^\circ \]
SOLUTIONS TO MECHANICAL ENGINEERING PROBLEMS

1. Heat is transferred from the primary system to the secondary system through thin U-shaped tubes. The water is heated to convert it to steam. The steam-water mixture rises into a moisture separator. The steam continues into the steam system. The liquid is returned to the region where the feed water enters the steam generator to preheat the feed water.

2. 

1) Attach hose to car and barge. Pump is not required as oil in the car will gravity drain to the barge.

2) Open barge vent

3) Open car vent

4) Open valves to hose on car and barge

3. The Rankine cycle is an ideal cycle since all its processes are reversible. It is used as an ideal model for steam power plants.

1-2: Constant pressure heating from subcooled liquid state to boiling to superheated vapor state

2-3: Adiabatic, isentropic expansion in turbine of superheated vapor into wet steam.

3-4: Constant pressure, constant temperature condensation in the condensor.

4-1: Adiabatic isentropic compression in pump.

The thermal efficiency is the net work \( q_s - q_r \) divided by the net heat supplied \( q_s \).

\[
\eta_t = \frac{q_s - q_r}{q_s}
\]

From the T-S diagram it can be seen that to increase efficiency the boiler pressure and temperature and superheat temperature should be as high as physically possible and the condensor pressure and temperature should be as low as is practical.

4. With parallel flow (solid lines) there is initially a large difference between fluid temperatures \( t_1 \) and \( t_2 \), with and attendant high rate of heat transfer in this part of the apparatus. But as flow continues through the
limiting value and the difference between fluid temperatures decreases rapidly, thereby decreasing heat transfer. With counterflow (dashed lines) the lowest and highest fluid temperatures are placed side by side at opposite ends of the apparatus. Therefore, the average temperature difference is greater, causing greater heat transfer. The exit temperature of the hot fluid can be made to approach the lowest temperature of the coolant, instead of only being able to approach the highest temperature of the coolant as is the case with parallel flow.

5.  Fuel to Combustion Chamber

1-2: Isentropic compression

2-3: Constant pressure heat addition

3-4: Isentropic expansion through Turbine

4-1: Exhaustion of working substance to atmosphere and intake of air.

6. Open inlet valve
   Vent pump
   Start pump, vent pump
   Open outlet valve
   Transfer water
   Shut outlet valve, secure pump
   Shut inlet valve

7. It is not, because of heat loss from primary piping and heat losses in the secondary system including excessive cooling in the condenser. The energy required to run the pumps must be subtracted from plant output.
1. Buckling ($B^2$) is related to the surface-to-volume ratio of a reactor. $B^2$ is larger for a wide, high, extremely thin slab than for a cube of equivalent volume. A large surface-to-volume ratio indicates a large probability of neutron leakage from the reactor.

For a homogeneous, bare parallelepiped core, buckling is

$$B^2 = \frac{\nu_x}{x^2} + \frac{\nu_y}{y^2} + \frac{\nu_z}{z^2}$$

thus $B^2$ decreases as the dimension along the $z$ axis increases. (for example: rod height increasing)

2. The pressurizer maintains reactor plant pressure at an elevated value. Elevated pressure allows the plant to operate at higher temperatures while avoiding boiling in the core. Higher plant temperatures yield higher steam pressures on the secondary side, giving more efficient performance in the steam plant.

The pressurizer performs its function by maintaining a steam bubble in the pressurizer. A steam bubble mitigates pressure changes due to plant coolant volume changes by providing a surge volume.

3. [Diagram of a nuclear reactor system with labeled components: shielding, steam, turbine, condenser, coolant pump, and heat exchanger.]
4. Fission – the splitting of a large mass nucleus into two lighter fragments with a release of energy.

All nucleons exert very short range attractive forces on one another. This force saturates, that is, one nucleon can attract only a limited number of other nucleons. All protons (positively charged) exert relatively long range repulsive forces on one another. The relative proportion of neutrons must increase to exert additional attractive forces as the atomic mass increases.

The nucleus continues to become less stable with increasing mass despite an increased neutron-to-proton ratio. The energy per nucleon holding the nucleus together becomes less. Critical energy for fission is supplied when the target nucleus absorbs a neutron.

One theory considers the excited compound nucleus (after neutron absorption) to oscillate like a liquid drop before fissioning into two fragments of unequal mass, with a release of excess neutrons. This allows the possibility of a chain reaction.

5. Criticality is a nuclear chain reaction where one neutron is produced for every neutron lost from the reactor by leakage, neutron capture or absorption by control rods.

The neutron multiplication factor are definitions which give a convenient method of analyzing the physical events that happen in a reactor.

The four factor formula

$$k_{\infty} = \varepsilon_{pnf}$$

$\varepsilon$ – fast fission factor

is the ratio of fast plus thermal fission rates to thermal fission rates in an infinite reactor

$\rho$ – resonance escape probability

$\rho$ is the fraction of fission neutrons born which will slow to thermal energies in an infinite reactor

$n$ – reproduction factor

$n$ is the number of fission neutrons produced per uranium-235 thermal absorption

$f$ – thermal utilization

$f$ is the fraction of thermalized neutrons that is absorbed in uranium-235 in an infinite reactor

6. Binding energy is the energy that would have to be supplied to break up a nucleus into the component neutrons and protons (plus their associated electrons). Equivalently, it is also the energy that would be released if a nucleus could be made from its component parts. When a heavy nucleus fissions the binding energy is released in the form of kinetic energy of fission fragments and neutrons, instantaneous gamma rays and gamma rays
from neutron capture. This energy is converted to thermal energy by collisions with structural materials and the coolant.

7. The negative temperature coefficient, \( \alpha_T \), predicts the change in the neutron multiplication factor, \( k_{\text{eff}} \), that occurs with a change in temperature.

As \( T \uparrow \), \( k_{\text{eff}} \) becomes less positive. A critical reactor becomes subcritical and a supercritical reactor moves toward criticality.

The neutron energy spectrum shifts to a higher level as \( T \uparrow \).

Microscopic thermal absorption cross sections decrease with increasing neutron energy, so that it is more probable for lower energy thermal neutrons to be absorbed than for higher energy thermal neutrons to be absorbed.

8. Fission is the process by which a heavy atom is split into lighter atoms releasing energy where as fusion is a process that joins lighter atoms forming heavier atoms releasing energy. A breeder reactor is a fission-type reactor that creates fuel for the fission process by a neutron absorption into a stable isotope of uranium to form plutonium.

9. A BWR has a single steam cycle in which the reactor core heats the feedwater to steam, which after being expanded through turbines is condensed and fed back to the core.

A PWR has a two cycle system where the primary system transport heat from the reactor core to a heat exchanger. This system is under high pressure to ensure no boiling occurs. The secondary systems removes heat from the primary through the heat exchanger operating the steam cycle to do work.
10. Neutron radiation is attenuated in shielding by elastic scattering. Hydrogen nuclei form the most effective attenuator due to their large macroscopic cross section for scattering.

The mass of the hydrogen nuclei is nearly equal to the mass of the neutron, so large losses of kinetic energy occur when a collision takes place effectively attenuating the neutron. Water, polyethylene, borated poly are good shielding because of low nuclide mass.

Gamma radiation is attenuated in shielding by several means:
- Photoelectric effect involves gamma interaction with an orbital electron. The electron moves to a higher energy shell or is removed completely, ionizing the atom. The gamma is annihilated.

- Compton scattering involves gamma interaction with an orbital electron. The result is a lower energy gamma and an electron moved to a higher energy shell or completely from the atom.

- Pair production results when a gamma of at least 1.02MeV interacts in the vicinity of a heavy nucleus to produce one electron and one positron. The electron loses energy by ionization. The positron interacts with other electrons and loses energy by ionizing them. At a low enough energy the positron combines with an electron, mutual annihilation occurs and the energy is released as a gamma. Steel and lead are good shielding because of relatively heavy mass.
1. 1) \( Q = U + W \) or the change in internal energy is equal to the quantity of heat added minus the work done by the system.

2) \( 0 < \frac{dQ}{T} < 0 \) or it is impossible to construct an engine that while operating in a cycle, will produce no effect other than the extraction of heat from a source and a conversion of this heat completely into work.

3) If the entropy of each element in some crystalline state = 0 at absolute zero, then every substance has a finite, positive entropy; but the entropy may become zero, at absolute zero in the case of perfect crystalline structures.

4) If A is in thermal equilibrium with B and C, then B is in thermal equilibrium with C.

2. 

1-2 Adiabatic reversible compression
2-3 Isothermal expansion
3-4 Adiabatic reversible expansion
4-1 Isothermal compression

The Carnot cycle is the most efficient cycle that can operate between two heat reservoirs.

1-2 Adiabatic comp to boiler pressure
2-3 Isobaric heating to boiling point
3-4 Isobaric, isothermal vaporization into saturated steam
4-5 Isobaric superheating
5-6 Adiabatic expansion into wet steam
6-1 Isobaric, isothermal condensation

The Rankine cycle is the cycle incorporating a phase change - for example, the steam cycle.
3. Use Fourier's Law for a one-dimensional slab. Assume exterior of bar to be in thermal equilibrium with its surroundings.

\[ Q = -kA \left( \frac{dt}{dx} \right) \]

where: \( Q \) is the constant rate of heat flow
\( k \) is the average thermal conductivity of the material
\( A \) is the cross-sectional area perpendicular to heat flow

\[ \frac{Q}{A} \int_0^L dx = -\int_a^T k \, dt \]

\[ Q = k \, A \, \frac{(T_a - T_b)}{L} \]

4. Enthalpy is the heat absorbed by a system at constant pressure with only work being done in expansion.

or

\[ H = U + PV \]

\[ \Delta H = \Delta U + \int PdV \]

5. Entropy is the measure of the irreversibility of a process. It indicates the efficiency of the process. Machine "b" would be preferred as its lower entropy change implies less heat lost to the environment and thus greater efficiency.

6. The addition of heat will increase the internal energy of the gas creating a temperature increase. By the first law, with constant volume, pressure will increase proportionally with temperature \((PV = nRT)\)

7. Yes, it is possible to have different pressures if the number of moles of gas is different. \( PV = nRT \)

8. English Units

\[ Q = mcp \, t + (100 \, \text{lb})(1 \, \text{BTU} / ^\circ\text{F} - 1\text{lb})(23^\circ\text{F}) \]

\[ = 2300 \, \text{BTU} \]

Metric Units

\[ m = 100 \, \text{lb}(454)/1\,\text{lb} = 4.54 \times 10^4 \, \text{g} \]

\[ t = 23\text{F}(5\text{C})/9F = 115\text{C}/9 = 12.8\text{C} \]
\[ Q = m c_p t = (4.5 \times 10^4 \text{ g})(1 \text{ cal.g}^{-1})(12.8 \text{C}) \]
\[ = 5.8 \times 10^5 \text{ cal} \]

9. Temperature is the physical property that determines if two systems are in thermal equilibrium.
   Heat content is a function of the quantity of heat transferred which depends on internal energy and work done.

10. Assumptions:
    1) temperature constant throughout procedure
    2) gases are inert to each other
    3) ideal gas behavior - low pressure, high temperature

\[ PV = nRT \]
\[ h_T = h_A + h_B + h_C \]
\[ \frac{P V_T}{RT} = \frac{P_A V_A}{RT} + \frac{P_B V_B}{RT} + \frac{P_C V_C}{RT} \]
\[ V_T = V_A + V_B + V_C \]
\[ P_T = P_A + P_B + P_C \]

OR by Dalton's law of partial pressures
\[ P_A = x_A P_T, \quad P_B = x_B P_T, \quad P_C = x_C P_T \]
where \( x \) is the molar fraction.
\[ x_A + x_B + x_C = 1 \]
\[ \frac{P_A}{P_T} + \frac{P_B}{P_T} + \frac{P_C}{P_T} = 1 \]
\[ P_T = P_A + P_B + P_C \]

11. The triple point is the pressure and temperature at which ice, liquid and water vapor coexist in equilibrium (273.16 K)

The region of five ices is to the left and above the triple point in the solid area. \( \Theta \) is the critical point to the right of which the vapor and liquid change phase to a gas.

12. Helmholtz Free Energy is useful in crystal energy mechanics and is constant for a chemical process that is reversible, isothermal and isochoric. \( F=U-TS \)

Gibbs Free Energy is useful in sublimation, fusion, and vaporization process analysis and is constant for reversible, isothermal and isobaric processes. \( G=H-TS \)

13. \( W = \int F \cdot dr \)
\[ W = W_1 \Delta h \]
14. Amount of mixing of the fluid, which converts mechanical pressure energy into heat energy, increasing the temperature along the pipe axis. Heat transfer through pipe walls, causing temperature variations along the axis of the pipe.
Thermocouple inaccuracies

Reference junctions not at same temperature calibration
Changing temperatures and delay time
Voltage source regulation
Inadequate heat transfer from the medium to the thermocouple

15. a) The process is reversible but does not exist in nature.
b) Since the entropy decreases during the process, it is fictitious.
c) Process 1 is more efficient than process 2.

16. Temperature will increase proportionally with pressure (PV = nRT). The heat addition will increase the internal energy of the gas. By the first law of thermodynamics and a given constant volume, pressure must increase.

17. $\int_{1}^{2} \frac{Q}{T} \, ds = 0$

18. 1) The entropy is a function of the thermodynamic coordinates whose change is equal to the integral of $\frac{\partial Q}{T}$ between the terminal states, integrated along any reversible path connecting the two states $\int_{1}^{2} \frac{\partial Q}{T}

2) Enthalpy is the sum of internal energy and pressure times volume (H=U+PV). The use of this property allows a function of three variables to be evaluated quickly.

3) Free energy (Helmholtz) $F = U - TS$ and F is constant for a reversible, isothermal and isochoric process (crystalline motions)

(Gibbs) $G = H - TS$ and G is constant for a reversible, isothermal and isobaric process (sublimation, fusion, vaporization).
4) Enthalpy can be measured in an isobaric process as a quantity of heat transferred. \( H_f - H_i = \int_C p \, dt \)

5) Entropy change can be calculated from the enthalpy change:

\[ \int_S - \int_I = \int_C \frac{dH}{T} \] for an isobaric process

19. Assumptions:
1) Fluid flow is ideal so there is no temperature increase from head loss.
2) The power output of the heater is independent of flow rate.
3) There is no heat transfer through the pipe walls.

\[ T_2 - T_1 = \frac{\dot{Q}}{C_p \dot{m}} = \frac{\dot{Q}}{(C_p A \cdot f \cdot V)} \]

where: \( Q, A, f, \) and \( C_p \) are constant

Therefore, the change in temperature across the heater is inversely proportional to the flow rate. Doubling the flow rate decreases the temperature increase by 50%.

20. The heat which flows through the solid must also pass through the high and low temperature surface films:

\[ \dot{Q}_{film1} = \dot{Q}_{solid} = \dot{Q}_{film2} \]

From Fourier's Law, heat transfer through a solid (conductive) is:

\[ \dot{Q}_{solid} = k(t_a - t_b)A_s/L \]

where: \( k \), the coefficient of heat transfer is assumed to be constant.

\( A_s \) is the surface area of the wall

Heat transfer through a fluid film is:

\[ \dot{Q}_{film} = h A_s(t_{high} - t_{low}) \]

Solve for the three temperature differences:

\[ t_1 - t_a = \frac{\dot{Q}}{(A_s h_1)} \]

\[ t_a - t_b = \frac{(\dot{Q} L)}{(A_s k)} \]
\[ t_b - t_2 = \frac{Q}{A_s h_2} \]

By addition:
\[ t_1 - t_2 = \frac{\dot{Q}}{A_s h_1} + \frac{L \dot{Q}}{A_s k} + \frac{\dot{Q}}{A_s h_2} \]
\[ \dot{Q} = A_s (t_1 - t_2)/(1/h_1 + L/k + 1/h_2) \]

21. Conduction—Heat transfer by conduction occurs when heat energy moves through a material as a result of the collisions between the molecules of the material. The efficiency in which a material can transfer heat is proportional to its thermal conductivity (k).

\[ \dot{Q} = k(T_1 - T_2)A/L \]

Convection—This mode occurs when a warm material is transported so as to displace a cooler material. Mass transfer must occur. Typical examples are the flow of warm air from a register in a heating system and the flow of cool water past a piece of hot metal.

\[ \dot{Q} = h A(T_1 - T_2) \]

where: \( h \), the film coefficient, is the rate of heat transfer of a particular fluid for a particular condition of surface turbulence.

Radiation—The transport of heat energy between separated bodies through intervening space by electromagnetic wave motion. The radiation from a solid or liquid surface is:

\[ \dot{Q}_R = \sigma A e T^4 \]

where: \( \sigma \) = the Stephan-Boltzmann constant
\( T \) = absolute temperature of the body
\( e \) = emissivity of the body

22.

Diagram:

- **HEAT OF VAPORIZATION** (
  \( 540 \text{ cal/g} \))

- **HEAT OF FUSION** (
  \( 80 \text{ cal/g} \))

\( 0^\circ C \) to 100\(^\circ\)C
CHEMISTRY SOLUTIONS

1. $3 \text{C}_3\text{H}_3 + \text{O}_2 \longrightarrow \text{CO}_2 + \text{H}_2\text{O}$
   
   Balancing yields
   
   $3\text{C}_3\text{H}_3 + 11\frac{1}{4} \text{O}_2 \longrightarrow 9 \text{CO}_2 + 9/2 \text{H}_2\text{O}$
   
   .'. 9 moles CO$_2$ from burning 3 moles C$_3$H$_3$

2. $\text{pH} = -\log[\text{H}_3\text{O}^+]$
   
   pH of pure water = 7
   
   pH decreases as hydrogen ion concentration increases.
   
   $K_{\text{H}_2\text{O}} = 10^{-14} = [\text{H}_3\text{O}^+][\text{OH}^-]$
   
   pH decreases with increasing temperature.

3. Radioactive decay: rate of decay = amount present
   
   $-\frac{dA}{dt} = -Kd$t
   
   $\frac{dA}{A} = -Kdt$
   
   $\ln A = -Kt + C$
   
   $e^{\ln A} = e^{-Kt + C} = e^{-Kt}e^C$
   
   $A = (e^{-Kt})(A_0)$ where $e^C = A = A_0$ when $t = 0$
   
   $A = A_0e^{-Kt}$

4. Add several drops of AgNO$_3$. If Cl$^-$ is present, a milky white precipitate (AgCl) will form.

5. Assuming the body is 100% water and water is 18 g/mole,
   
   # of molecules = (100 lbs.)($454$ g/lb.)(1 mole/18 g)(6.023 x 10$^{23}$ molecules/mole)
   
   $= 1.51 \times 10^{27}$ molecules

6. If the reaction requires heat (solution is cooled), it is endothermic; if it releases heat, it is exothermic.
7. There is covalent bonding between the nitrogen and hydrogen atoms.

8. \[ N_L = N_0 \left( 1 - e^{-at} \right) = N_0 \left( 1 - e^{-\left(\frac{.693(T/2)}{40\text{yrs.}}\right)} \right) \]

\[ = 1 - e^{-1.04} \]

\[ = 1 - .35 \]

\[ N_L/N_0 = .65 \]

\[ \text{.} 65\% \text{ had decayed.} \]

9. \[
\begin{array}{ccc}
\text{4 L pH 4} & [\text{H}_3\text{O}^+] & [\text{OH}^-] \\
4 \times 10^{-4} \text{ moles} & 4 \times 10^{-10} \text{ moles} \\
\text{6 L pH 6} & 6 \times 10^{-6} \text{ moles} & 6 \times 10^{-8} \text{ moles} \\
10 \text{ L} & 4.06 \times 10^{-4} \text{ moles} & 6.04 \times 10^{-8} \text{ moles} \\
\end{array}
\]

Result: \[ [\text{H}_3\text{O}^+] = 4.0 \times 10^{-5} \text{ moles/L} \]

\[ = 10^{-.60} \times 10^{-5} \]

\[ = 10^{-4.4} \]

\[ \text{pH} = 4.4 \]

10. Valence is a positive or negative integer value that indicates the ratio in which hydrogen or oxygen will combine with it. It is a function of the number of electrons available or missing in an outer shell of the atom in an element state. These excesses or vacancies are removed or filled, creating ions.

11. Diffusion of \( \text{CO}_2 \) carbonic acid lowers pH by 1.5 units; \( \text{H}_2\text{O} \) evaporation.

12. Ionic Bonding is a type of chemical combination between two or more atoms to form a permanent electric dipole. The atoms are joined through the relationship of outer shell electrons. When the bond is broken, positive and negative ions are formed.
13. The electron affinities of the ions involved in a reaction determines the spontaneity of the reaction.

14. Corrosion is the oxidization of a metal by exposure to the atmosphere, i.e., the metal ions give up their excess electrons as they form a compound with oxygen molecules.

15. An acid is a solution with a molar concentration of the hydronium ion, $H_3O^+$, of greater than $10^{-7}$ moles/liter.

16. Covalent bonds are involved, and the molecule doesn't fly apart because each atom has a full outer shell.

17. $2H = 2g$
    $1S = 32g$
    $4O = 64g$

    $100g \times \left( \frac{1\text{ mole/98g}}{} \right) \left( 6.023 \times 10^{23} \text{ molecules/mole} \right) \left( 7\text{ atoms/molecule} \right)$

    $= 4.302 \times 10^{24}$ atoms of $H_2SO_4$

18. The substance is flashed to steam by routing it through an area of low pressure, thereby removing impurities.

19. $Fe^{2+} + 2Cl^- + 2H_3O^+ \rightarrow FeCl_2 + 2H_3O^+$

    $\frac{1\text{ mole } Fe^{2+}}{} \times 3\text{ moles } HCl = 1.5 \text{ moles } Fe^{2+}$

    $1.5 \text{ moles } Fe^{2+} \left( 55.85g/mole \right) = 83.8 \text{ grams}$

20. A buffer solution is a solution in which the $[H^+]$ and pH are not appreciably affected by the addition of small amounts of acids and bases; usually a buffer solution will contain relatively large amounts of both a weak acid and a weak base.